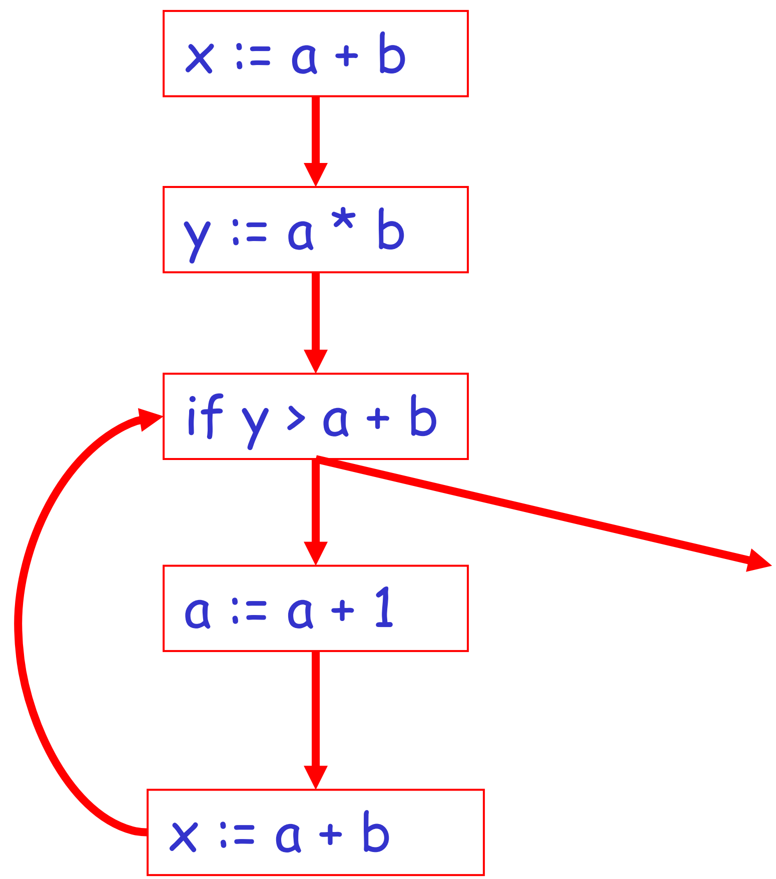
Introduction to Dataflow Analysis

Control-Flow Graphs

```
x := a + b;
y := a * b;
while y > a + b {
   a := a + 1;
   x := a + b
}
```



Notation

s is a statement

succ(s) = { *successor statements of s* }

pred(s) = { *predecessor statements of s* }

write(s) = { *variables written by s* }

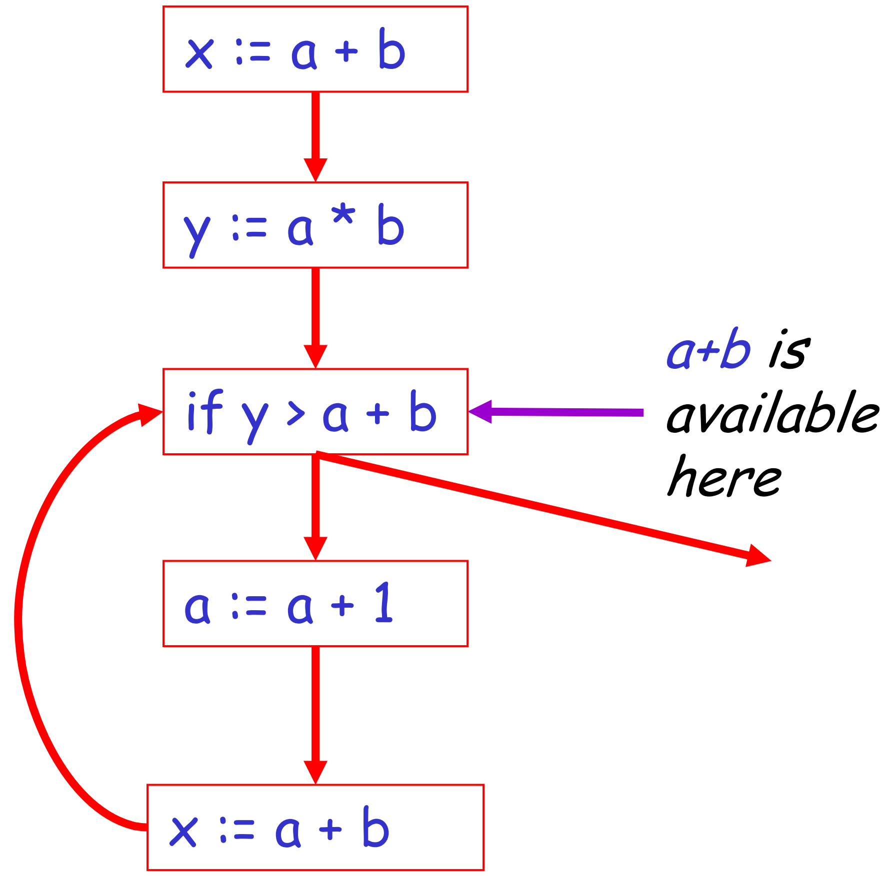
read(s) = { *variables read by s* }

Note: In literature write = kill and read = gen

Available Expressions

- For each program point p, finds which expressions must have already been computed, and not later modified, on all paths to p.

- Optimization: Where available, expressions need not be recomputed.

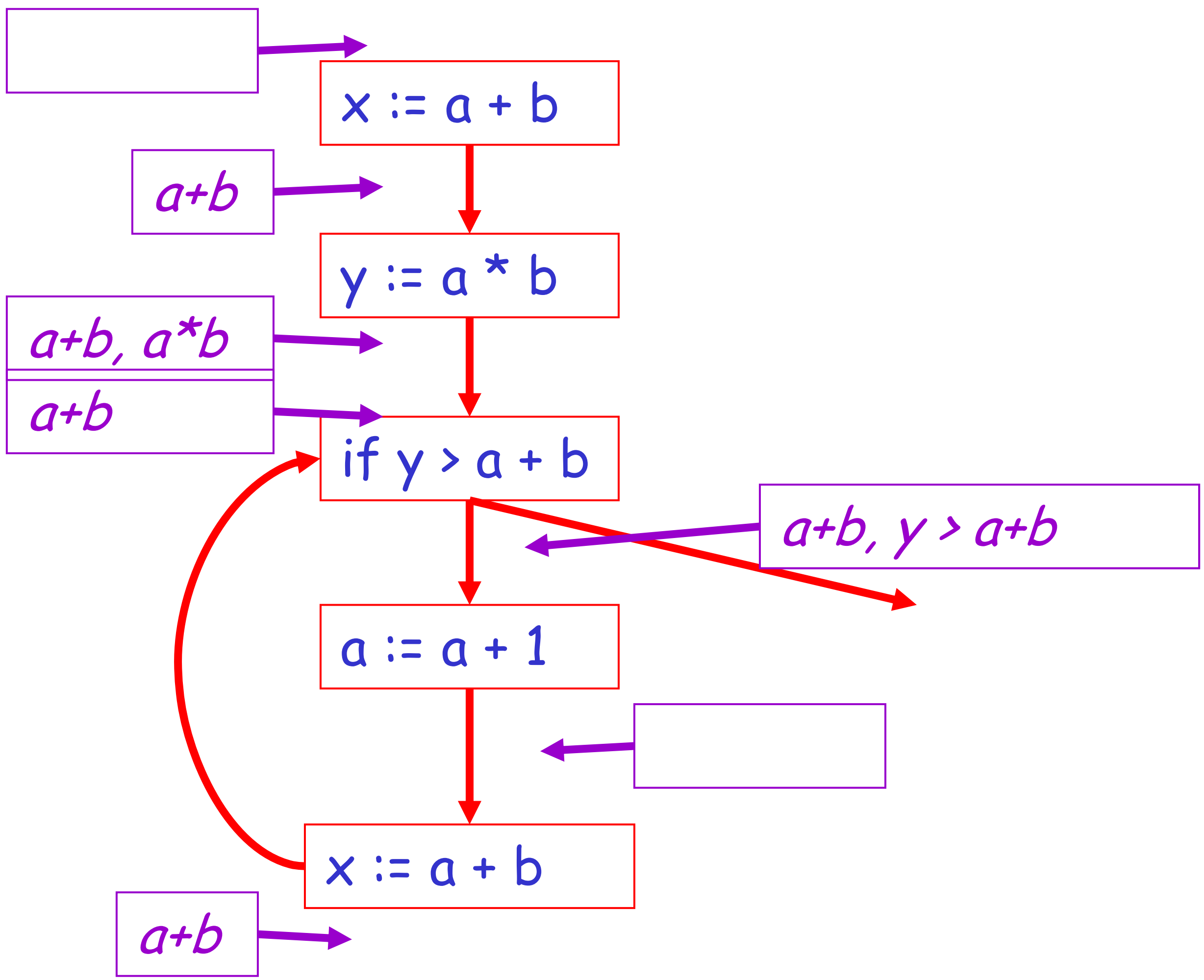


Dataflow Equations

$$A_{in}(s) = \begin{cases} \emptyset & \text{if } pred(s) = \emptyset \\ \bigcap_{s' \in pred(s)} A_{out}(s') & \text{otherwise} \end{cases}$$

$$A_{out}(s) = (A_{in}(s) - \{a \in S \mid write(s) \cap V(a) \neq \emptyset\})$$
$$\cup \{s \mid \text{if } write(s) \cap read(s) = \emptyset\}$$

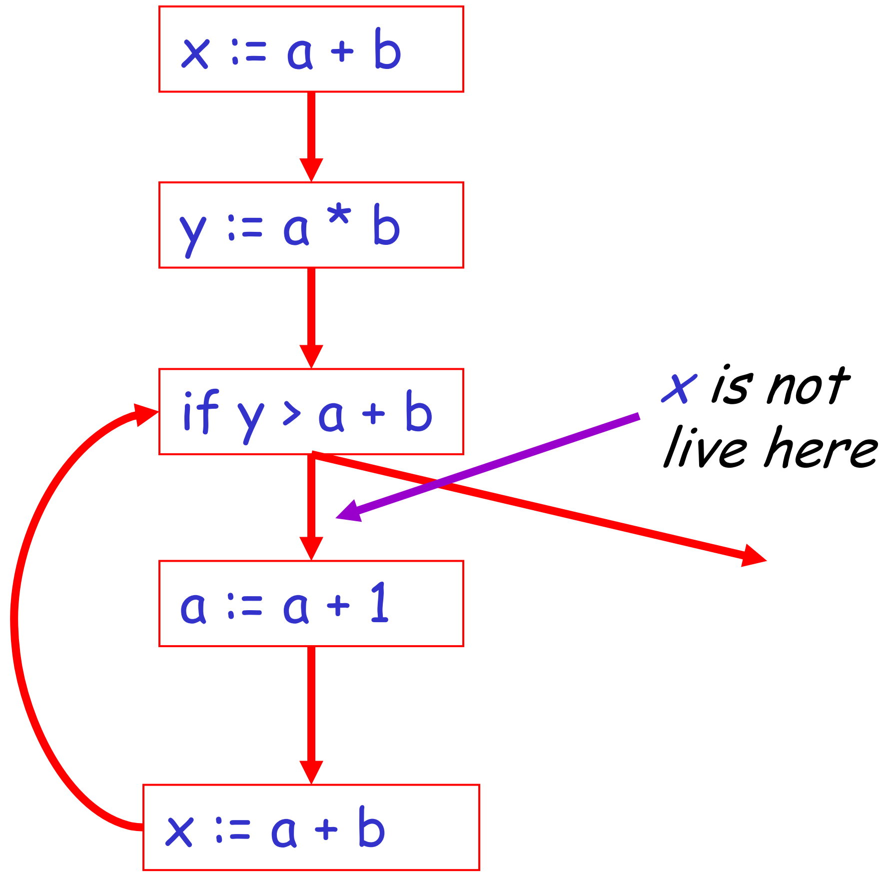
Example



Liveness Analysis

- For each program point p, finds which of the variables defined at that point are used on some execution path?

- Optimization: If a variable is not live, there is no need to keep it in a register.

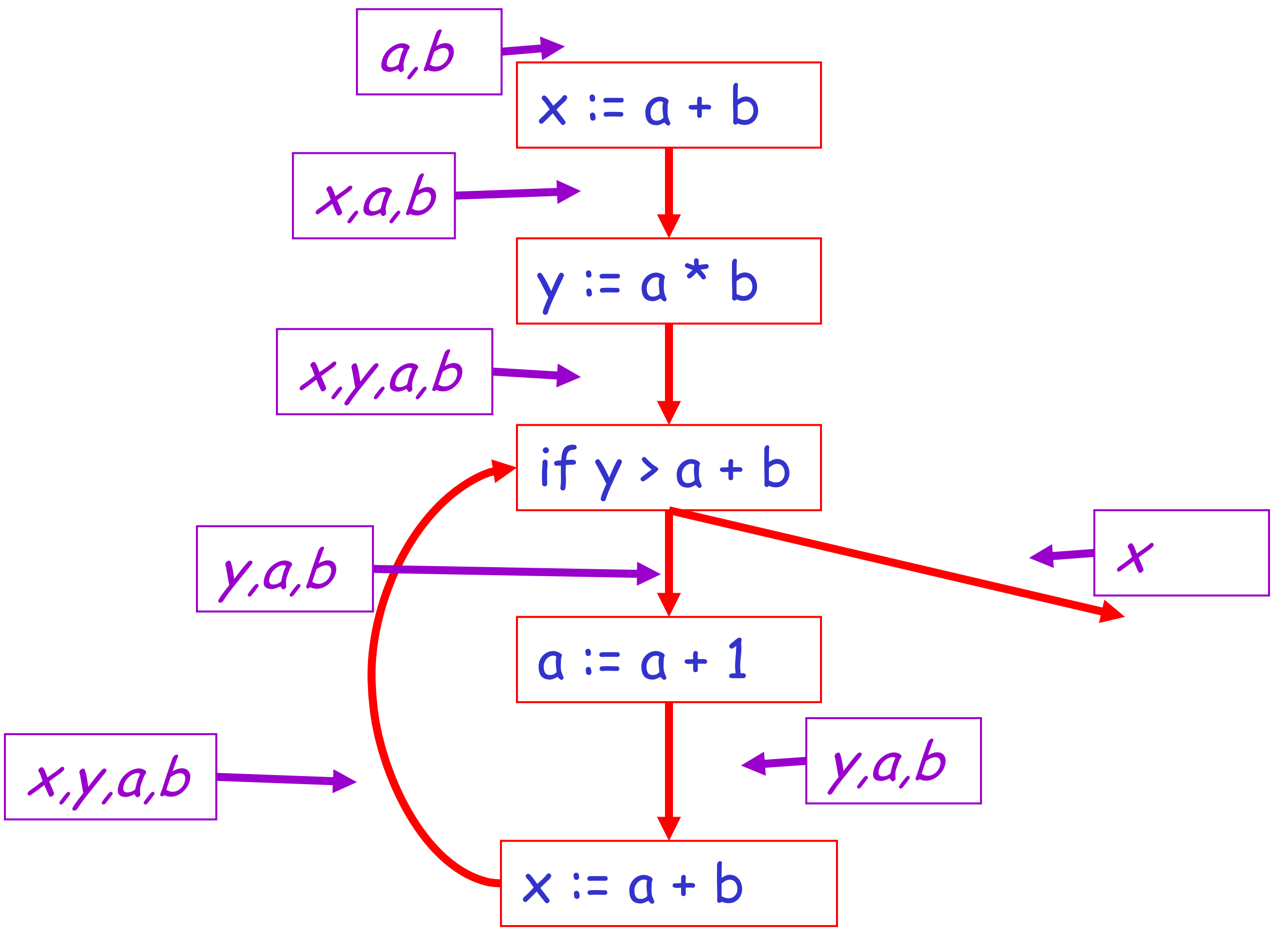


Dataflow Equations

$$L_{in}(s) = (L_{out}(s) - write(s)) \cup read(s)$$

$$L_{out}(s) = \left\{ \begin{array}{cc} \emptyset & \text{if } succ(s) = \emptyset \\ \bigcup_{s' \in succ(s)} L_{in}(s') & \text{otherwise} \end{array} \right\}$$

Example



Available Expressions Again

$$A_{in}(s) = \begin{cases} \varnothing & \text{if } pred(s) = \varnothing \\ \bigcap_{s' \in pred(s)} A_{out}(s') & \text{otherwise} \end{cases}$$

$$A_{out}(s) = (A_{in}(s) - \{a \in S \mid write(s) \cap V(a) \neq \varnothing\})$$
$$\cup \{s \mid write(s) \cap read(s) = \varnothing\}$$

Available Expressions: Schematic

$$A_{in}(s) = \bigcap_{s' \in pred(s)} A_{out}(s')$$

Transfer function:

$$A_{out}(s) = A_{in}(s) - C_1 \cup C_2$$

Must analysis: property holds on all paths

Forwards analysis: from inputs to outputs

Live Variables Again

$$L_{in}(s) = (L_{out}(s) - write(s)) \cup read(s)$$

$$L_{out}(s) = \left\{ \begin{array}{cc} \varnothing & \text{if } succ(s) = \varnothing \\ \bigcup_{s' \in succ(s)} L_{in}(s') & \text{otherwise} \end{array} \right\}$$

Live Variables: Schematic

Transfer function:

$$L_{in}(s) = L_{out}(s) - C_1 \cup C_2$$

$$L_{out}(s) = \bigcup_{s' \in succ(s)} L_{in}(s')$$

May analysis: property holds on some path

Backwards analysis: from outputs to inputs

Very Busy Expressions

- An expression *e* is very busy at a program point p if every path from p must evaluate e before any variable in e is redefined

- Optimization: hoisting expressions

- A must-analysis
- A backwards analysis

Reaching Definitions

- For a program point p, which assignments made on paths reaching p have not been overwritten
- Connects definitions with uses (use-def chains)
- A may-analysis
- A forwards analysis

One Cut at the Dataflow Design Space

	May	*Must*
Forwards	Reaching definitions	Available expressions
Backwards	Live variables	Very busy expressions

The Literature

- **Vast literature of dataflow analyses**

- **90+% can be described by**
  - Forwards or backwards
  - May or must

- **Some oddballs, but not many**
  - Bidirectional analyses

Flow Sensitivity

- **Flow sensitive analyses**
  - The order of statements matters
  - Need a control flow graph
    - Or transition system, ...
- **Flow insensitive analyses**
  - The order of statements doesn't matter
  - Analysis is the same regardless of statement order

Example Flow Insensitive Analysis

- **What variables does a program fragment modify?**

$$G(x := e) = \{x\}$$

$$G(s_1; s_2) = G(s_1) \cup G(s_2)$$

- Note $G(s_1;s_2) = G(s_2;s_1)$

The Advantage

- **Flow-sensitive analyses require a model of program state at each program point**
  - E.g., liveness analysis, reaching definitions, …

- **Flow-insensitive analyses require only a single global state**
  - E.g., for *G*, the set of all variables modified

Notes on Flow Sensitivity

- **Flow insensitive analyses seem weak, but:**
- **Flow sensitive analyses are hard to scale to very large programs**
  - Additional cost: state size X # of program points
- **Beyond 1000's of lines of code, only flow insensitive analyses have been shown to scale**

Context-Sensitive Analysis

- **What about analyzing across procedure boundaries?**

```
Def f(x){…}
Def g(y){…f(a)…}
Def h(z){…f(b)…}
```

- **Goal:** Specialize analysis of f to take advantage of
  - f is called with a by g
  - f is called with b by h

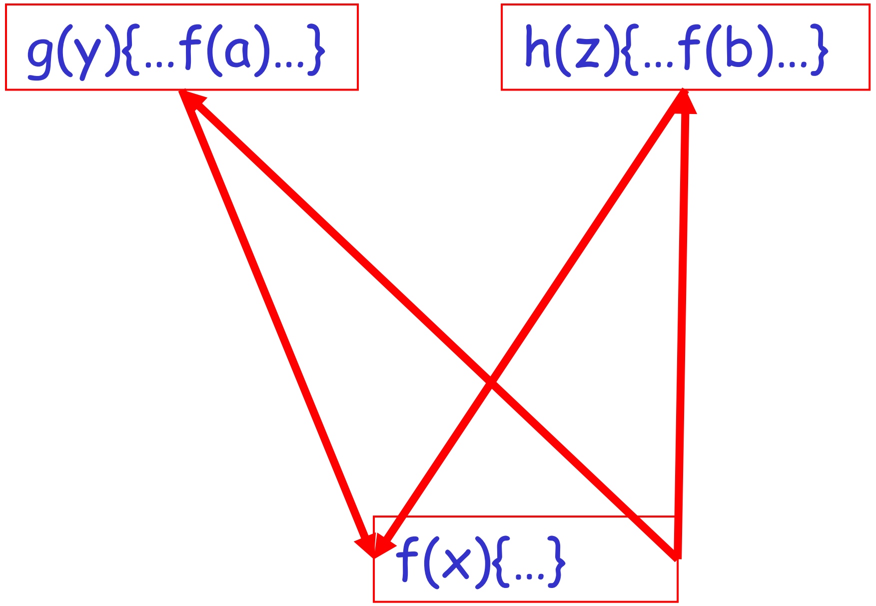
Control-Flow Graphs Again

- **How do we extend control-flow graphs to procedures?**
- **Idea: Model procedure call f(a) by:**
  - Edge from point before call to entry of f
  - Edge from exit(s) of f to point after call

Example

Edges from

- Before f(a) to entry of f
- Exit of f to after f(a)
- Before f(b) to entry of f
- Exit of f to after f(b)

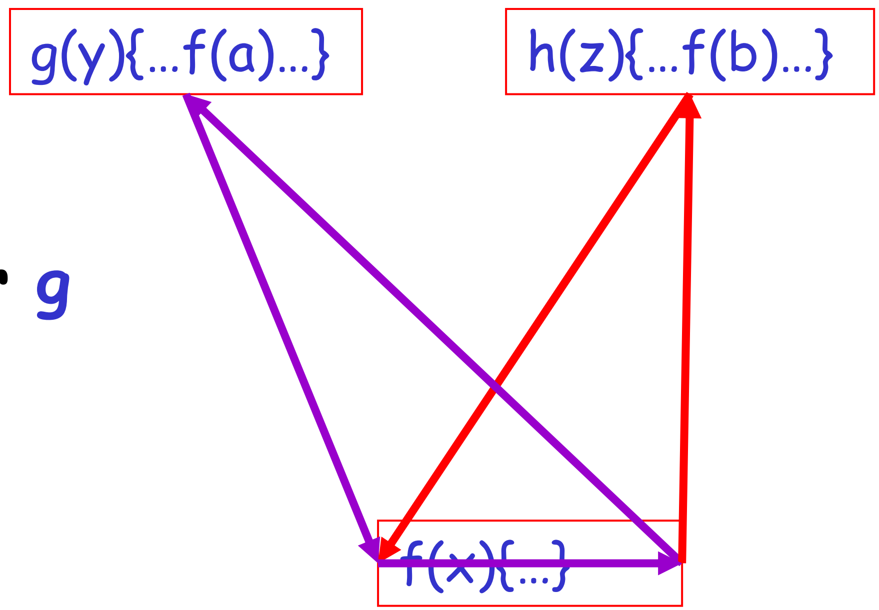


Example

Edges from

- Before f(a) to entry of f
- Exit of f to after f(a)
- Before f(b) to entry of f
- Exit of f to after f(b)

Has the correct flows for g

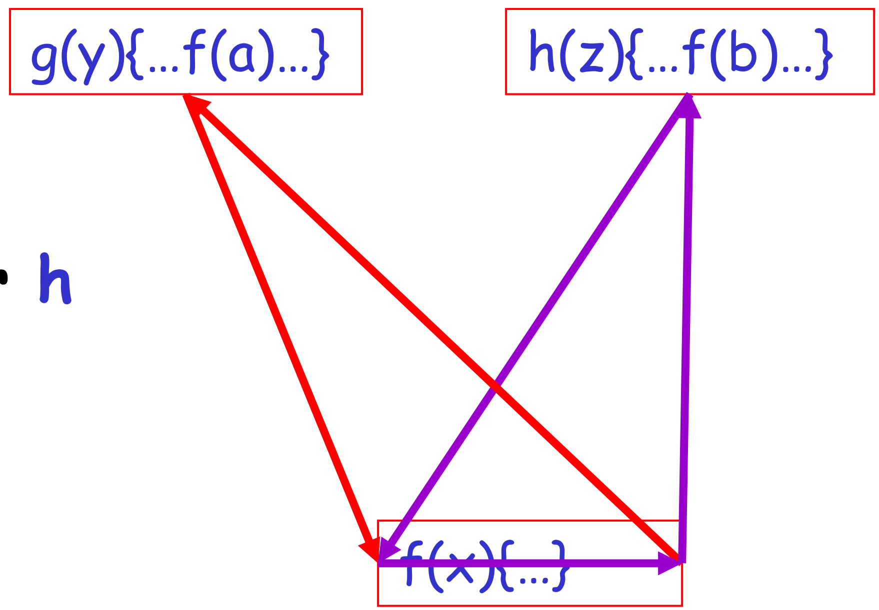


Example

Edges from

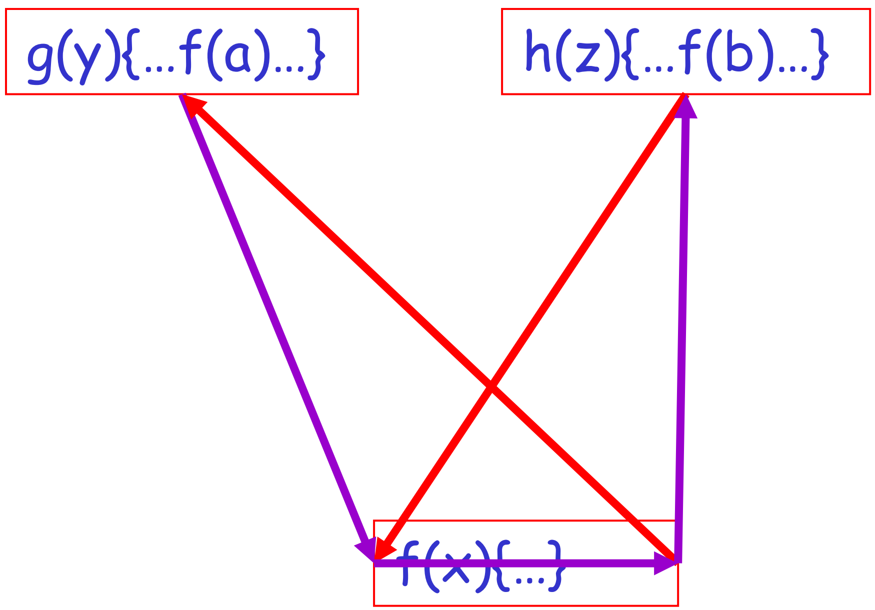
- Before f(a) to entry of f
- Exit of f to after f(a)
- Before f(b) to entry of f
- Exit of f to after f(b)

Has the correct flows for h



Example

- **But also has flows we don't want**
  - One path captures a call to g returning at h!
- **So-called "infeasible paths"**
- **Must distinguish calls to f in different contexts**



Review of Terminology

- Must vs. May
- Forwards vs. Backwards
- Flow-sensitive vs. Flow-insensitive
- Context-sensitive vs. Context-insensitive

Where is Dataflow Analysis Useful?

- **Best for flow-sensitive, context-insensitive problems on small pieces of code**
  - E.g., the examples we've seen and many others
- **Extremely efficient algorithms are known**
  - Use different representation than control-flow graph, but not fundamentally different
  - More on this in a minute . . .

Where is Dataflow Analysis Weak?

- Lots of places

Data Structures

- **Not good at analyzing data structures**
- **Works well for atomic values**
  - Labels, constants, variable names
- **Not easily extended to arrays, lists, trees, etc.**
  - Work on shape analysis

The Heap

- **Good at analyzing flow of values in local variables**
- **No notion of the heap in traditional dataflow applications**
- **In general, very hard to model anonymous values accurately**
  - Aliasing
  - The "strong update" problem

Context Sensitivity

- Standard dataflow techniques for handling context sensitivity don't scale well
- Brittle under common program edits

Flow Sensitivity (Beyond Procedures)

- **Flow sensitive analyses are standard for analyzing single procedures**
- **Not used (or not aware of uses) for whole programs**
  - Too expensive

The Call Graph

- **Dataflow analysis requires a call graph**
  - Or something close
- **Inadequate for higher-order programs**
  - First class functions
  - Object-oriented languages with dynamic dispatch
- **Call-graph hinders algorithmic efficiency**
  - Desire to keep executable specification is limiting

Forwards vs. Backwards

- **Restriction to forwards/backwards reachability**
  - Very constraining
  - Many important problems not easy to fit into this mold