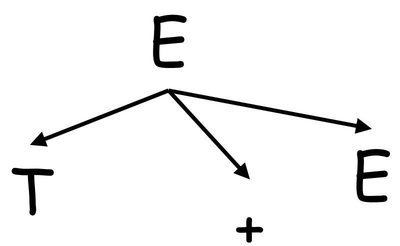
Introduction to Bottom-Up Parsing

Outline

- Review LL parsing
- Shift-reduce parsing
- The LR parsing algorithm
- Using LR parsing tables

Top-Down Parsing: Review

- Top-down parsing expands a parse tree from the start symbol to the leaves.
  - Always expand the leftmost non-terminal.



int * int + int

$E \rightarrow T + E \mid T$

$T \rightarrow (E) \mid int \mid int * T$

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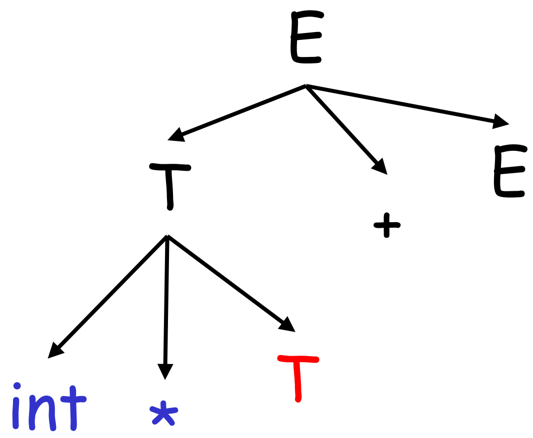

int * int + int

- The leaves at any point form a string $\beta A \gamma$, where
  - β contains only terminals.
  - The input string is $\beta b \delta$.
  - The prefix β matches.
  - The next token is b.

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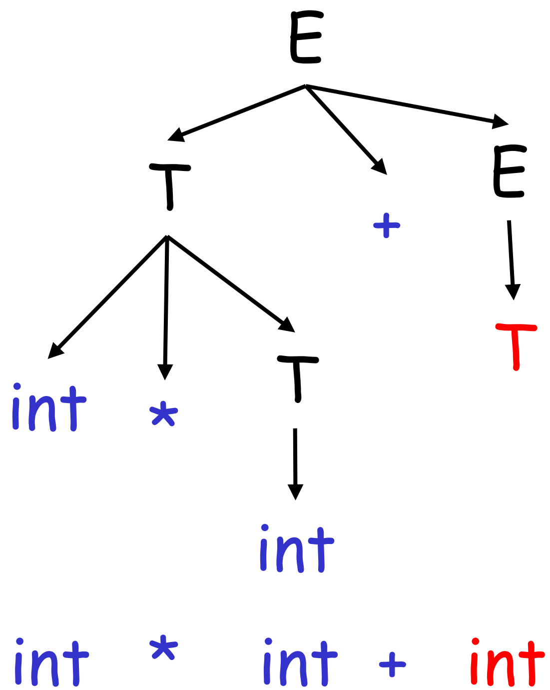

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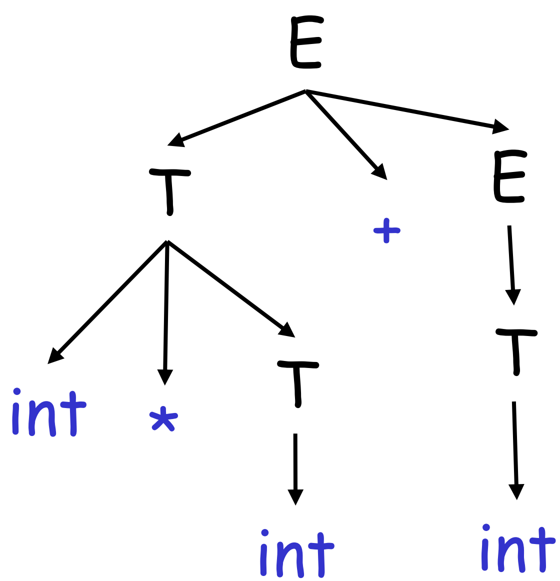

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  - The prefix β matches.
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$E \rightarrow T + E \mid T$

$T \rightarrow (E) \mid \text{int} \mid \text{int} * T$

Predictive Parsing: Review

- A predictive parser is described by a table.
  - For each non-terminal A and for each token b we specify a production $A \to \alpha$.
  - When trying to expand A we use $A \to \alpha$ if b is the token that follows next.
- Once we have the table:
  - The parsing algorithm is simple and fast.
  - No backtracking is necessary.

Constructing Predictive Parsing Tables

Consider the state $S \to^* \beta A \gamma$

- With b the next token
- Trying to match $\beta b \delta$

There are two possibilities:

1. Token b belongs to an expansion of A
    - Any $A \to \alpha$ can be used if b can start a string derived from α
    - We say that $b \in First(\alpha)$

Or…

Constructing Predictive Parsing Tables (Cont.)

2. Token b does not belong to an expansion of A
   - The expansion of A is empty and b belongs to an expansion of γ
   - Means that b can appear after A in a derivation of the form $S \rightarrow^* \beta A b \omega$
   - We say that $b \in Follow(A)$ in this case

   - What productions can we use in this case?
     - Any $A \rightarrow \alpha$ can be used if α can expand to ε
     - We say that $\varepsilon \in First(A)$ in this case

Computing First Sets

<u>Definition</u>

$$\text{First}(X) = \{\, b \mid X \to^* b\alpha \,\} \cup \{\, \varepsilon \mid X \to^* \varepsilon \,\}$$

<u>Algorithm sketch</u>

1. $\text{First}(b) = \{\, b \,\}$
2. $\varepsilon \in \text{First}(X)$ if $X \to \varepsilon$ is a production
3. $\varepsilon \in \text{First}(X)$ if $X \to A_1 \ldots A_n$ and $\varepsilon \in \text{First}(A_i)$ for $1 \le i \le n$
4. $\text{First}(\alpha) \subseteq \text{First}(X)$ if $X \to A_1 \ldots A_n\, \alpha$ and $\varepsilon \in \text{First}(A_i)$ for $1 \le i \le n$

First Sets: Example

- Recall the grammar

$E \to T\ X$ $\qquad$ $X \to +\ E \mid \varepsilon$

$T \to (\ E\ ) \mid \text{int}\ Y$ $\qquad$ $Y \to *\ T \mid \varepsilon$

- First sets

First(( ) = { (} $\qquad$ First(T) = { int, (}

First( )) = {) } $\qquad$ First(E) = { int, (}

First(int) = { int } $\qquad$ First(X) = { +, ε }

First(+) = { + } $\qquad$ First(Y) = { *, ε }

First(*) = { * }

Computing Follow Sets

- Definition

$$\text{Follow}(X) = \{ b \mid S \to^* \beta\, X\, b\, \delta \}$$

- Intuition
  - If $X \to A\ B$ then $\text{First}(B) \subseteq \text{Follow}(A)$
    and $\text{Follow}(X) \subseteq \text{Follow}(B)$
  - Also if $B \to^* \varepsilon$ then $\text{Follow}(X) \subseteq \text{Follow}(A)$
  - If S is the start symbol then $\$ \in \text{Follow}(S)$

Computing Follow Sets (Cont.)

Algorithm sketch

1. $\$ \in \text{Follow}(S)$
2. $\text{First}(\beta) - \{\varepsilon\} \subseteq \text{Follow}(X)$
   - For each production $A \to \alpha\ X\ \beta$
3. $\text{Follow}(A) \subseteq \text{Follow}(X)$
   - For each production $A \to \alpha\ X\ \beta$ where $\varepsilon \in \text{First}(\beta)$

Follow Sets: Example

First(T) = { int, (}
First(E) = { int, (}
First(X) = { +, ε }
First(Y) = { *, ε }

- Recall the grammar

$E \to T\ X$ $\qquad X \to +\ E \mid \varepsilon$

$T \to (\ E\ ) \mid int\ Y$ $\qquad Y \to *\ T \mid \varepsilon$

- Follow sets

Follow(+) = { int, (}
Follow(*) = { int, (}
Follow(( ) = { int, (}
Follow(E) = {), $ }
Follow(X) = {), $ }
Follow(T) = { +,) , $ }
Follow( )) = { +,) , $ }
Follow(Y) = { +,) , $ }
Follow(int) = { *, +,) , $ }

Constructing LL(1) Parsing Tables

- Construct a parsing table T for CFG G.

- For each production $A \rightarrow \alpha$ in G do:
  - For each terminal $b \in \text{First}(\alpha)$ do
    $T[A, b] = \alpha$
  - If $\varepsilon \in \text{First}(\alpha)$, for each $b \in \text{Follow}(A)$ do
    $T[A, b] = \alpha$
  - If $\varepsilon \in \text{First}(\alpha)$ and $\$ \in \text{Follow}(A)$ do
    $T[A, \$] = \alpha$

Constructing LL(1) Tables: Example

- Recall the grammar

$E \to T\,X$ $\qquad\qquad$ $X \to +\,E \mid \varepsilon$

$T \to (\,E\,) \mid \text{int}\ Y$ $\qquad\qquad$ $Y \to *\,T \mid \varepsilon$

- Where in the line of Y do we put $Y \to *\,T$?
  - In the lines of First($*T$) = { $*$ }

- Where in the line of Y do we put $Y \to \varepsilon$?
  - In the lines of Follow(Y) = { $\$$, $+$, $)$ }

Notes on LL(1) Parsing Tables

- If any entry is multiply defined then G is not LL(1). This happens:
  - if G is ambiguous;
  - if G is left recursive;
  - if G is not left-factored;
  - <u>and in other cases as well.</u>

- For some grammars there is a simple parsing strategy: *Predictive parsing.*
- Most programming language grammars are not LL(1).
- Thus, we need more powerful parsing strategies.

Bottom Up Parsing

Bottom-Up Parsing

- Bottom-up parsing is more general than top-down parsing – and just as efficient.
  - Builds on ideas in top-down parsing.
  - Preferred method in practice.

- Also called LR parsing
  - L means that tokens are read left-to-right.
  - R means that it constructs a rightmost derivation.

An Introductory Example

- LR parsers don't need left-factored grammars and can also handle left-recursive grammars.

- Consider the following grammar:

$$E \rightarrow E + (E) \mid \text{int}$$

  - Why is this not LL(1)?

- Consider the string: int + (int) + (int)

The Idea

- LR parsing *reduces* a string to the start symbol by inverting productions.

str w input string of terminals

repeat

- Identify β in str such that $A \rightarrow \beta$ is a production (i.e., str = $\alpha\ \beta\ \gamma$)
- Replace β by A in str (i.e., str w = $\alpha\ A\ \gamma$)

until str = S (the start symbol)

OR all possibilities are exhausted

A Bottom-up Parse in Detail (1)

$E \rightarrow E + (E) \mid int$

int + (int) + (int)

int + (int) + (int)

A Bottom-up Parse in Detail (2)

$E \rightarrow E + (E) \mid \text{int}$

int + (int) + (int)

E + (int) + (int)

E

int + (int) + (int)

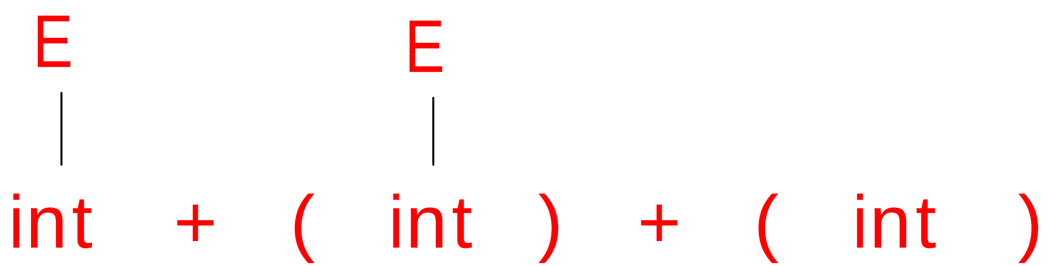
A Bottom-up Parse in Detail (3)

$E \rightarrow E + (E) \mid \text{int}$

int + (int) + (int)

E + (int) + (int)

E + (E) + (int)



A Bottom-up Parse in Detail (4)

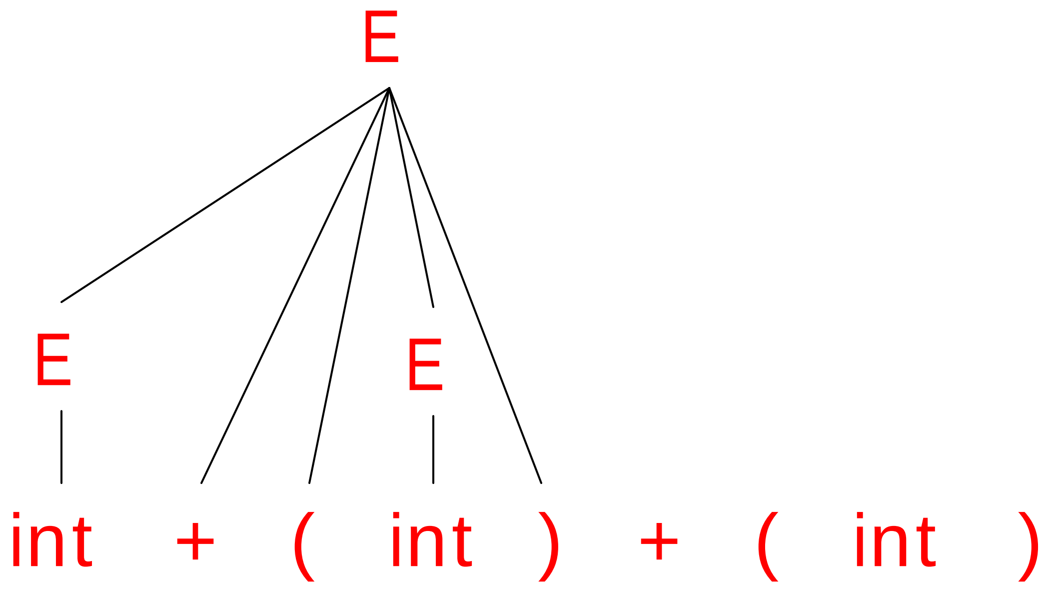
$E \rightarrow E + (E) \mid int$

int + (int) + (int)

E + (int) + (int)

E + (E) + (int)

E + (int)



A Bottom-up Parse in Detail (5)

$E \rightarrow E + (E) \mid int$

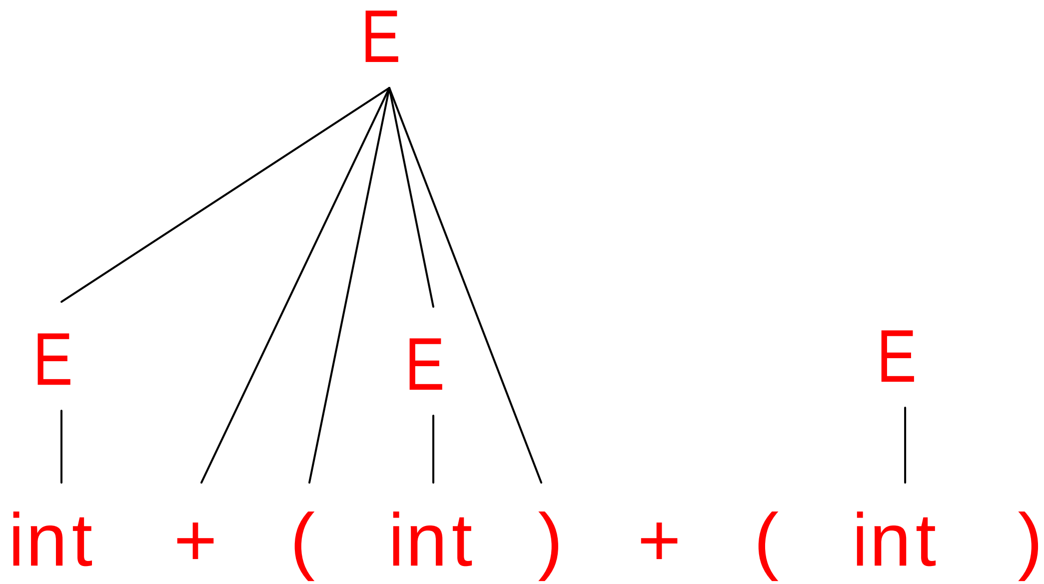
int + (int) + (int)

E + (int) + (int)

E + (E) + (int)

E + (int)

E + (E)

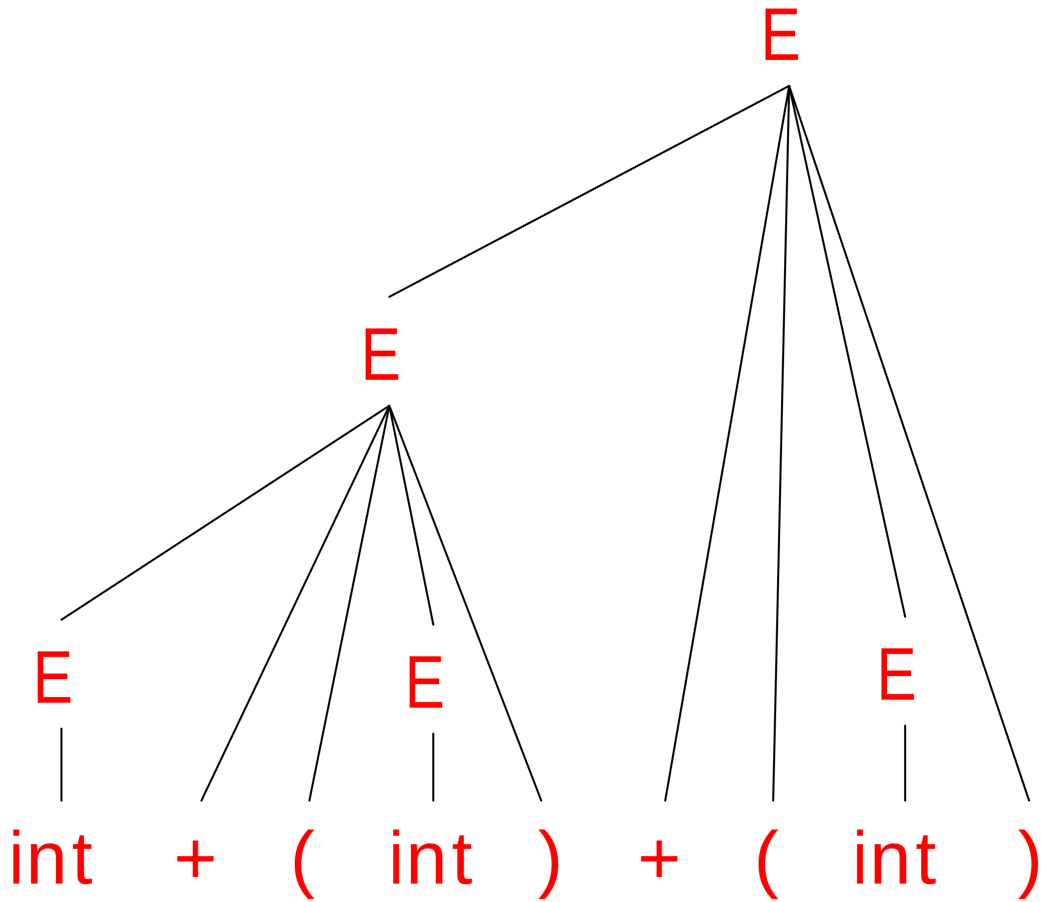


A Bottom-up Parse in Detail (6)

$E \rightarrow E + (E) \mid \text{int}$

int + (int) + (int)
E + (int) + (int)
E + (E) + (int)
E + (int)
E + (E)
E

A rightmost derivation in reverse



Important Fact #1 about Bottom-up Parsing

An LR parser traces a rightmost derivation in reverse.

Where Do Reductions Happen

Fact #1 has an interesting consequence:

- Let $\alpha\beta\gamma$ be a string of non-terminals and terminals of a bottom-up parse.
- Assume the next reduction is by using $A \rightarrow \beta$.
- Then γ is a string of terminals.

Why?

Because $\alpha A\gamma \rightarrow \alpha\beta\gamma$ is a step in a right-most derivation.

Notation

- Idea: Split string into two substrings:
  - Right substring is as yet unexamined by parsing (a string of terminals).
  - Left substring has terminals and non-terminals.

- The dividing point is marked by a $\mid$
  - The $\mid$ is not part of the string.

- Initially, all input is unexamined: $\mid x_1 x_2 \ldots x_n$

Shift-Reduce Parsing

Bottom-up parsing uses only two kinds of actions:

Shift

Reduce

Shift

Shift: Move I one place to the right.

- Shifts a terminal to the left string.

$$E + (\mid \text{int}\ ) \Rightarrow E + (\text{int} \mid)$$

In general:

$$ABC \mid xyz \Rightarrow ABCx \mid yz$$

Reduce

Reduce: Apply an inverse production at the right end of the left string.

If $E \to E + (E)$ is a production, then:

$$E + (\underline{E + (E)} \mid) \Rightarrow E + (\underline{E} \mid)$$

In general, given $A \to xy$, then:

$$Cbxy \mid ijk \Rightarrow CbA \mid ijk$$

Shift-Reduce Example

$E \rightarrow E + (E) \mid int$

I int + (int) + (int)$ shift

int + (int) + (int)

Shift-Reduce Example

$E \rightarrow E + (E) \mid int$

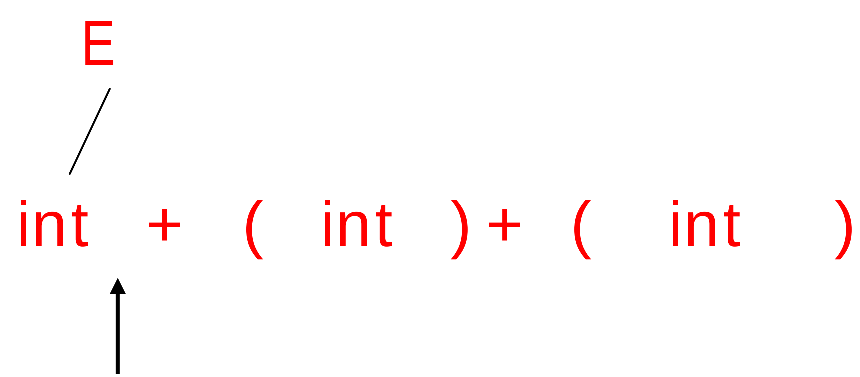
I int + (int) + (int)$	shift
int I + (int) + (int)$	reduce $E \rightarrow$ int

int + (int) + (int)

Shift-Reduce Example

$E \rightarrow E + (E) \mid int$

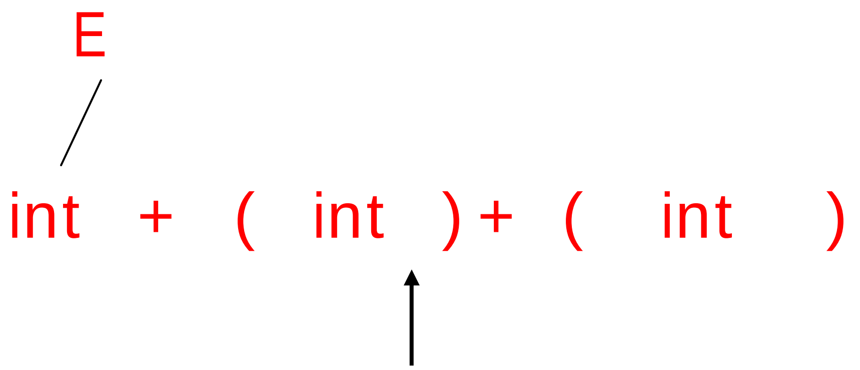
| int + (int) + (int)$	shift
int | + (int) + (int)$	reduce $E \rightarrow int$
E | + (int) + (int)$	shift 3 times



Shift-Reduce Example

$E \rightarrow E + (E) \mid int$

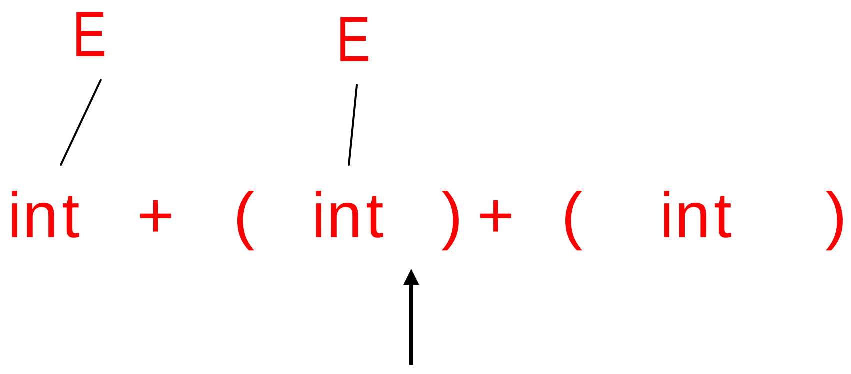
| int + (int) + (int)$	shift
int | + (int) + (int)$	reduce $E \rightarrow int$
E | + (int) + (int)$	shift 3 times
E + (int |) + (int)$	reduce $E \rightarrow int$



Shift-Reduce Example

E → E + (E) | int

| int + (int) + (int)$	shift
int | + (int) + (int)$	reduce E → int
E | + (int) + (int)$	shift 3 times
E + (int |) + (int)$	reduce E → int
E + (E |) + (int)$	shift



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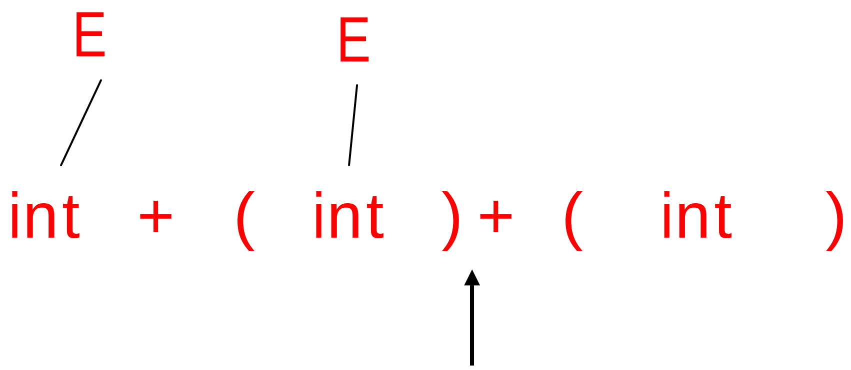
E + (int |) + (int)$	reduce $E \rightarrow int$
E + (E |) + (int)$	shift
E + (E) | + (int)$	reduce $E \rightarrow E + (E)$



Shift-Reduce Example

E → E + (E) | int

| int + (int) + (int)$ shift

int | + (int) + (int)$ reduce E → int

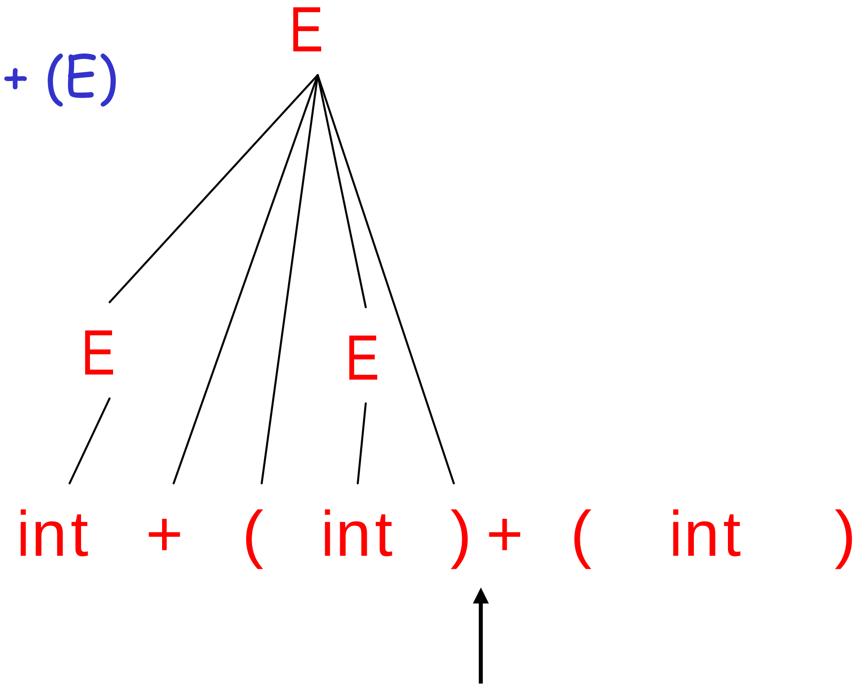
E | + (int) + (int)$ shift 3 times

E + (int |) + (int)$ reduce E → int

E + (E |) + (int)$ shift

E + (E) | + (int)$ reduce E → E + (E)

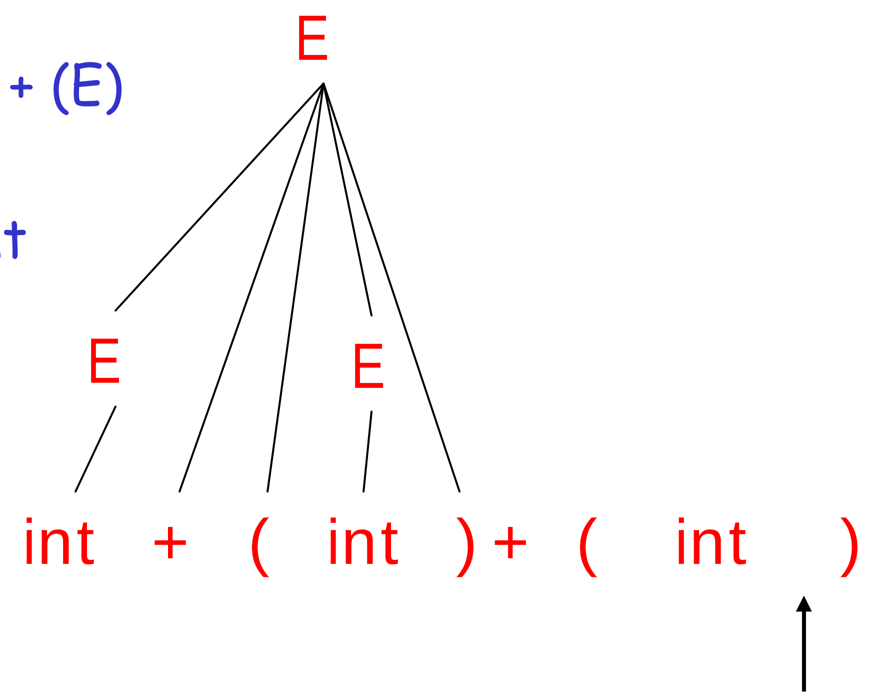
E | + (int)$ shift 3 times



Shift-Reduce Example

$E \rightarrow E + (E) \mid int$

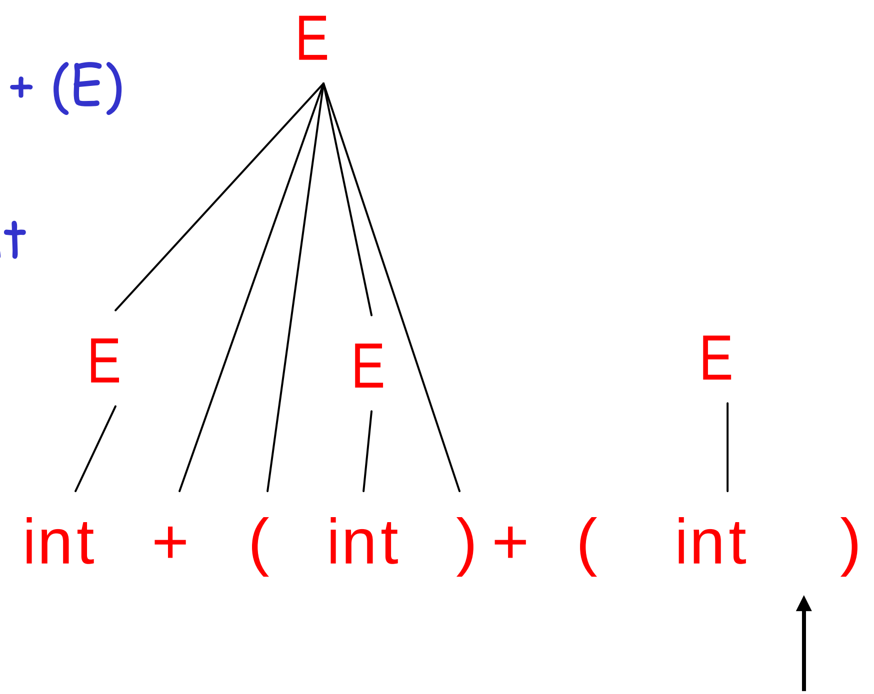
❘ int + (int) + (int)$	shift
int ❘ + (int) + (int)$	reduce $E \rightarrow int$
E ❘ + (int) + (int)$	shift 3 times
E + (int ❘) + (int)$	reduce $E \rightarrow int$
E + (E ❘) + (int)$	shift
E + (E) ❘ + (int)$	reduce $E \rightarrow E + (E)$
E ❘ + (int)$	shift 3 times
E + (int ❘)$	reduce $E \rightarrow int$



Shift-Reduce Example

$E \rightarrow E + (E) \mid int$

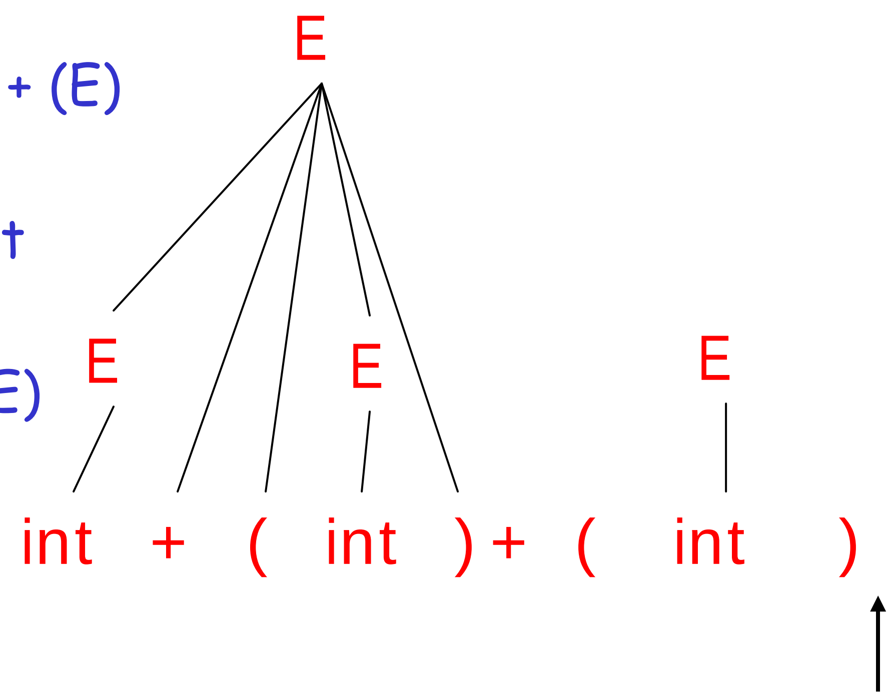
$\mid$ int + (int) + (int)$	shift
int $\mid$ + (int) + (int)$	reduce $E \rightarrow$ int
E $\mid$ + (int) + (int)$	shift 3 times
E + (int $\mid$) + (int)$	reduce $E \rightarrow$ int
E + (E $\mid$) + (int)$	shift
E + (E) $\mid$ + (int)$	reduce $E \rightarrow$ E + (E)
E $\mid$ + (int)$	shift 3 times
E + (int $\mid$)$	reduce $E \rightarrow$ int
E + (E $\mid$)$	shift



Shift-Reduce Example

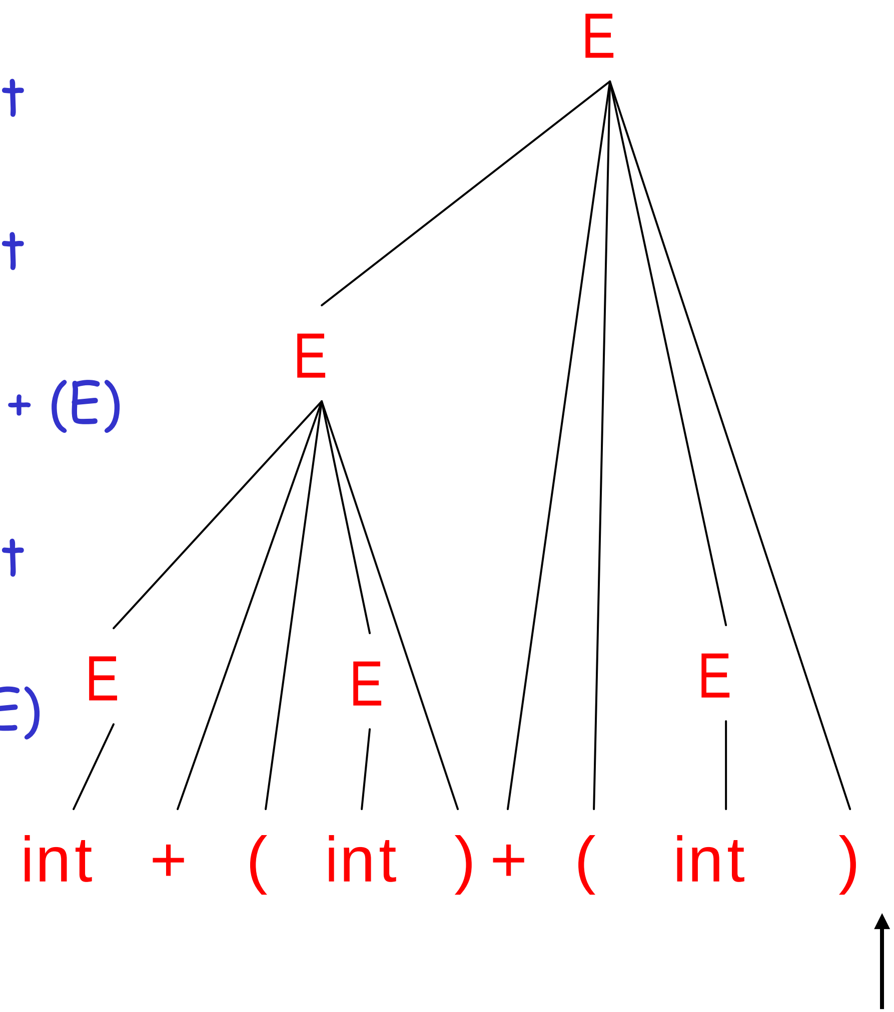
$E \rightarrow E + (E) \mid int$

| int + (int) + (int)$	shift
int | + (int) + (int)$	reduce E → int
E | + (int) + (int)$	shift 3 times
E + (int |) + (int)$	reduce E → int
E + (E |) + (int)$	shift
E + (E) | + (int)$	reduce E → E + (E)
E | + (int)$	shift 3 times
E + (int |)$	reduce E → int
E + (E |)$	shift
E + (E) | $	reduce E → E + (E)



Shift-Reduce Example

| int + (int) + (int)$	shift
int | + (int) + (int)$	reduce E → int
E | + (int) + (int)$	shift 3 times
E + (int |) + (int)$	reduce E → int
E + (E |) + (int)$	shift
E + (E) | + (int)$	reduce E → E + (E)
E | + (int)$	shift 3 times
E + (int |)$	reduce E → int
E + (E |)$	shift
E + (E) | $	reduce E → E + (E)
E | $	accept



The Stack

- Left string can be implemented by a stack.
  - Top of the stack is the |
- Shift pushes a terminal on the stack.
- Reduce pops 0 or more symbols off of the stack (production RHS) and pushes a non-terminal on the stack (production LHS).

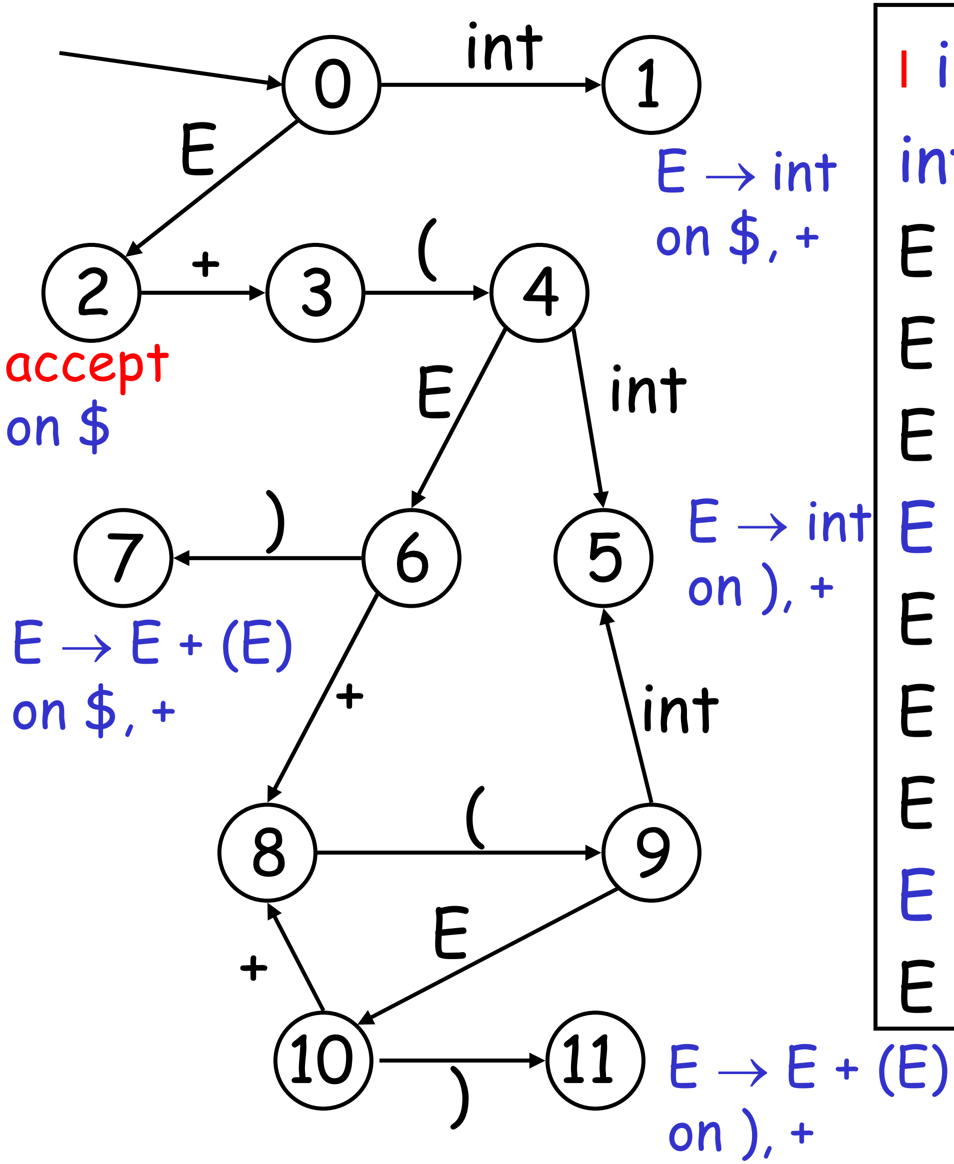
Key Question: To Shift or to Reduce?

<u>Idea</u>: use a finite automaton (DFA) to decide when to shift or reduce.

- The input is the stack.
- The language consists of terminals and non-terminals.

- We run the DFA on the stack and examine the resulting state X and the token tok after |
  - If X has a transition labeled tok then <u>shift</u>
  - If X is labeled with "$A \to \beta$ on tok" then <u>reduce</u>

LR(1) Parsing: An Example



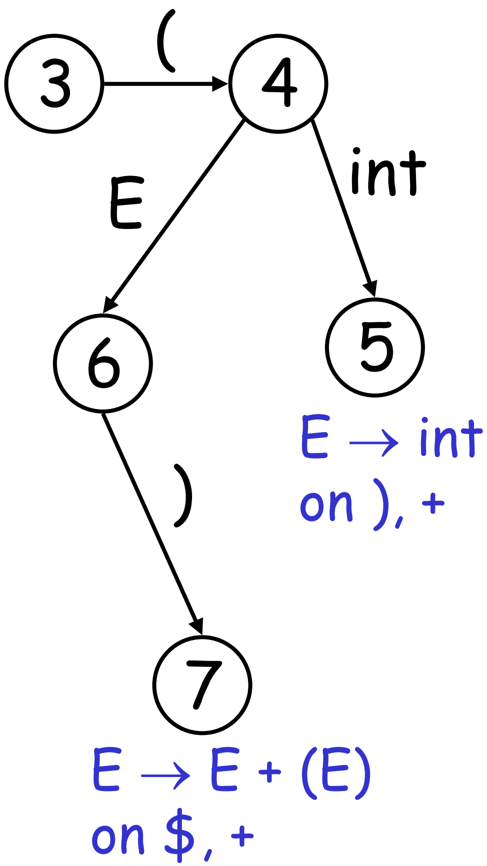
∣ int + (int) + (int)$	shift
int ∣ + (int) + (int)$	E → int
E ∣ + (int) + (int)$	shift(x3)
E + (int ∣) + (int)$	E → int
E + (E ∣) + (int)$	shift
E + (E) ∣ + (int)$	E → E+(E)
E ∣ + (int)$	shift (x3)
E + (int ∣)$	E → int
E + (E ∣)$	shift
E + (E) ∣ $	E → E+(E)
E ∣ $	accept

Representing the DFA

- Parsers represent the DFA as a 2D table.
  - (Recall table-driven lexical analysis)
- Lines correspond to DFA states.
- Columns correspond to terminals and non-terminals.
- Typically columns are split into:
  - Those for terminals: action table.
    - action = shift or reduce
  - Those for non-terminals: goto table.

Representing the DFA: Example

The table for a fragment of our DFA:



	int	+	(	)	$	E
…						
3			s4			
4	s5					g6
5		$r_{E \to \text{int}}$		$r_{E \to \text{int}}$		
6		s8		s7		
7		$r_{E \to E+(E)}$			$r_{E \to E+(E)}$	
…						

The LR Parsing Algorithm

- After a shift or reduce action we rerun the DFA on the entire stack.
  - This is wasteful, since most of the work is repeated.

- Remember for each stack element on which state it brings the DFA.

- LR parser maintains a stack

$$\langle\, sym_1, state_1 \,\rangle \;.\;.\;.\; \langle\, sym_n, state_n \,\rangle$$

$state_k$ is the final state of the DFA on $sym_1 \ldots sym_k$.

The LR Parsing Algorithm

```
let I = w$ be initial input
let j = 0
let DFA state 0 be the start state
let stack = ⟨ dummy, 0 ⟩
   repeat
     case action[top_state(stack), I[j]] of
       shift k: push ⟨ I[j++], k ⟩
       reduce X → A:
          pop |A| pairs,
          push ⟨X, Goto[top_state(stack),X]⟩
       accept: halt normally
       error: halt and report error
```

LR Parsers

- Can be used to parse more grammars than LL.
- Most programming languages grammars are LR.
- LR parsers can be described as a simple table.
- There are tools for building the table.

Next Lecture: How is the DFA constructed?