LR Parsing
LALR Parser Generators

Outline

- Review of bottom-up parsing
- Computing the parsing DFA
- Using parser generators

Bottom-up Parsing (Review)

- A bottom-up parser rewrites the input string to the start symbol.
- The state of the parser is described as:

$$\alpha \mid \gamma$$

  - α is a stack of terminals and non-terminals;
  - γ is the string of terminals not yet examined.

- Initially: $\mid x_1 x_2 \, . \, . \, . \, x_n$

The Shift and Reduce Actions (Review)

Recall the CFG: $E \rightarrow E + (E) \mid \text{int}$

A bottom-up parser uses two kinds of actions:

- Shift pushes a terminal from input on the stack

$$E + (\mid \text{int}) \Rightarrow E + (\text{int} \mid)$$

- Reduce pops 0 or more symbols off of the stack (production RHS) and pushes a non-terminal on the stack (production LHS)

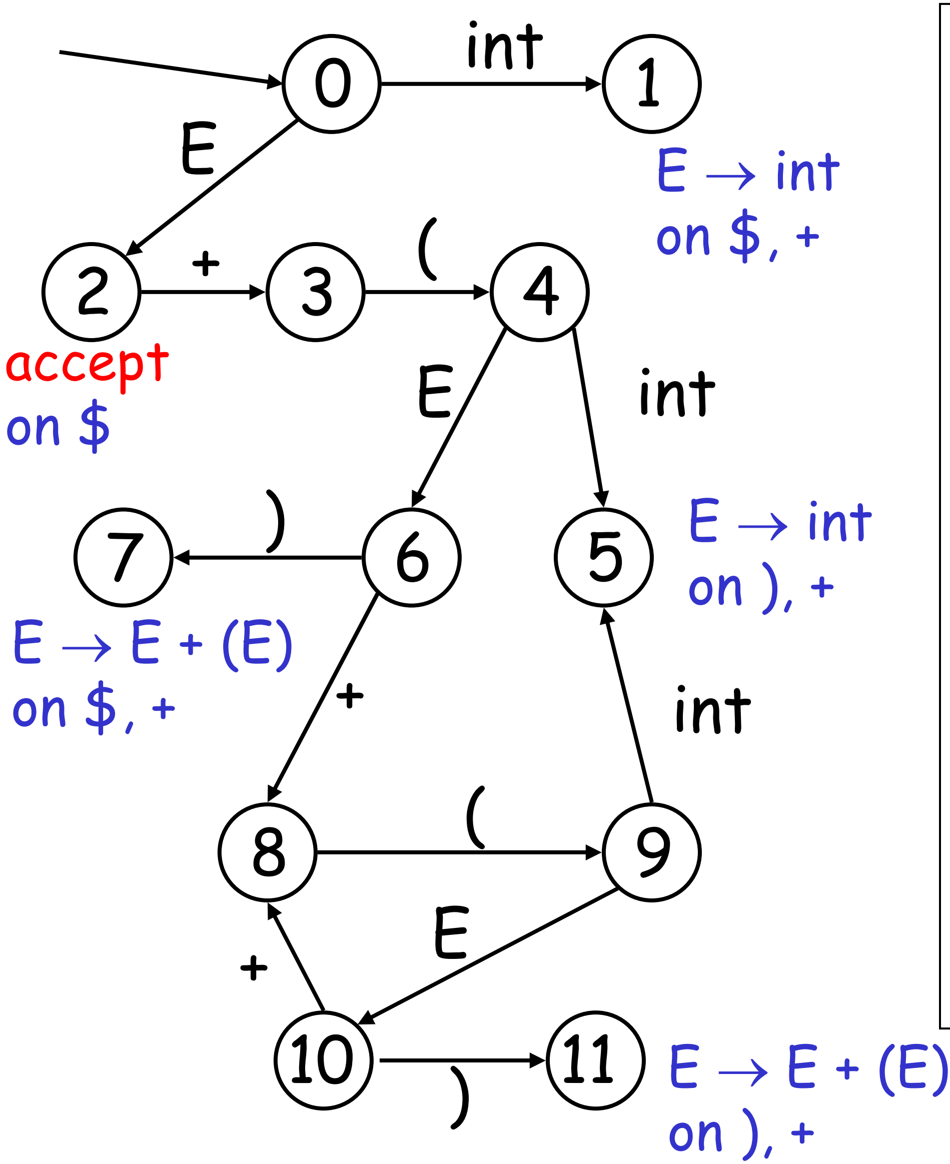
$$E + (\underline{E + (E)} \mid) \Rightarrow E + (\underline{E} \mid)$$

Key Issue: When to Shift or Reduce?

- Idea: use a deterministic finite automaton (DFA) to decide when to shift or reduce
  - The input is the stack
  - The language consists of terminals and non-terminals

- We run the DFA on the stack and we examine the resulting state X and the token tok after |
  - If X has a transition labeled tok then <u>shift</u>
  - If X is labeled with "$A \rightarrow \beta$ on tok" then <u>reduce</u>

LR(1) Parsing: An Example



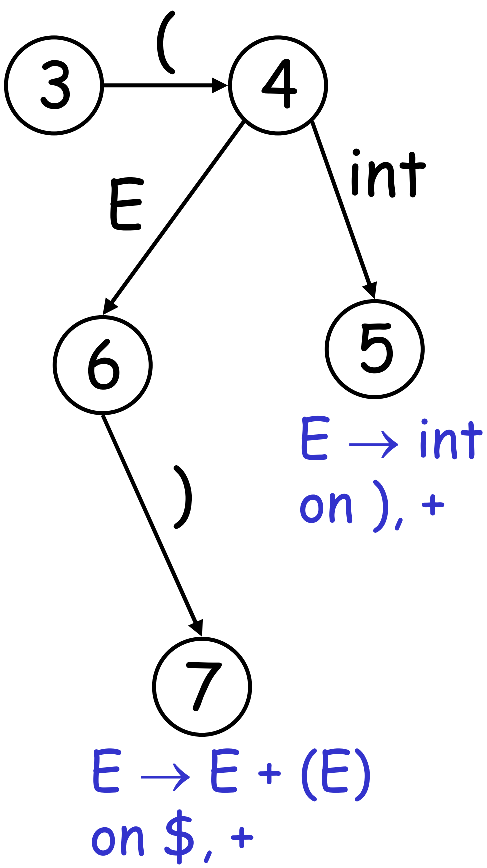
| int + (int) + (int)$	shift
int | + (int) + (int)$	E → int
E | + (int) + (int)$	shift (x3)
E + (int |) + (int)$	E → int
E + (E |) + (int)$	shift
E + (E) | + (int)$	E → E+(E)
E | + (int)$	shift (x3)
E + (int |)$	E → int
E + (E |)$	shift
E + (E) | $	E → E+(E)
E | $	accept

Representing the DFA

- Parsers represent the DFA as a 2D table.
  (Recall table-driven lexical analysis.)
- Lines correspond to DFA states.
- Columns correspond to terminals and non-terminals.
- Typically columns are split into:
  - Those for terminals: the action table.
  - Those for non-terminals: the goto table.

Representing the DFA: Example

The table for a fragment of our DFA:



	int	+	(	)	$	E
...						
3			s4			
4	s5					g6
5		$r_{E \rightarrow int}$		$r_{E \rightarrow int}$		
6		s8		s7		
7		$r_{E \rightarrow E+(E)}$			$r_{E \rightarrow E+(E)}$	
...						

sk is shift and goto state k
$r_{X \rightarrow \alpha}$ is reduce
gk is goto state k

The LR Parsing Algorithm

- After a shift or reduce action we rerun the DFA on the entire stack
  - This is wasteful, since most of the work is repeated

- To avoid this, we remember for each stack element on which state it brings the DFA.

- LR parser maintains a stack

$$\langle \text{sym}_1, \text{state}_1 \rangle \ldots \langle \text{sym}_n, \text{state}_n \rangle$$

state_k is the final state of the DFA on $\text{sym}_1 \ldots \text{sym}_k$

The LR Parsing Algorithm

```
let I = w$ be initial input
let j = 0
let DFA state 0 be the start state
let stack = ⟨ dummy, 0 ⟩
  repeat
    case action[top_state(stack), I[j]] of
      shift k: push ⟨ I[j++], k ⟩
      reduce X → A:
        pop |A| pairs,
        push ⟨ X, goto[top_state(stack), X] ⟩
      accept: halt normally
      error: halt and report error
```

Key Issue: How is the DFA Constructed?

- The stack describes the context of the parse:
  - What non-terminal we are looking for.
  - What production RHS we are looking for.
  - What we have seen so far from the RHS.

- Each DFA state describes several such contexts.

  E.g., when we are looking for non-terminal E, we might be looking either for an int or an E + (E) RHS.

LR(0) Items

- An <u>LR(0) item</u> is a production with a "I" somewhere on the RHS.

- The LR(0) items for $T \to (E)$ are

  $T \to \mathsf{I}\ (E)$

  $T \to (\ \mathsf{I}\ E)$

  $T \to (E\ \mathsf{I}\ )$

  $T \to (E)\ \mathsf{I}$

- The only LR(0) item for $X \to \varepsilon$ is $X \to \mathsf{I}$

LR(0) Items: Intuition

- An item $[X \to \alpha \mid \beta]$ says that the parser:
  - is looking for an X
  - has an α on top of the stack
  - expects to find a string derived from β next in the input.

- Notes:
  - $[X \to \alpha \mid a\beta]$ means that a should follow.
    - Then we can shift it and still have a viable prefix.
  - $[X \to \alpha \mid]$ means that we could reduce X.
    - But this is not always a good idea !

LR(1) Items

- An <u>LR(1) item</u> is a pair:

$$X \rightarrow \alpha \centerdot \beta, \ a$$

  - $X \rightarrow \alpha\beta$ is a production.
  - a is a terminal (the lookahead terminal).
  - LR(1) means 1 lookahead terminal.
- $[X \rightarrow \alpha \centerdot \beta, a]$ describes a context of the parser.
  - We are trying to find an X followed by an a.
  - We have (at least) α already on top of the stack.
  - Thus we need to see next a prefix derived from βa.

Note

- The symbol $\mid$ was used before to separate the stack from the rest of input:

  $\alpha \mid \gamma$, where α is the stack and γ is the remaining string of terminals.

- In items, $\mid$ is used to mark a prefix of a production RHS:

$$X \to \alpha \mid \beta, \ a$$

  - Here β might contain non-terminals as well.
- In either case, the stack is on the left of $\mid$

Convention

- We add to our grammar a fresh new start symbol S and a production $S \to E$
  - Where E is the old start symbol.

- The initial parsing context contains:

$$S \to \mid E \ , \$$$

  - Trying to find an S as a string derived from $E\$$
  - The stack is empty.

LR(1) Items (Cont.)

- In context containing

$$E \rightarrow E + \mid (E) \ , +$$

  - If (follows then we can perform a shift to context containing

$$E \rightarrow E + (\mid E) \ , +$$

- In context containing

$$E \rightarrow E + (E) \mid \ , +$$

  - We can perform a reduction with $E \rightarrow E + (E)$
  - But only if a + follows

LR(1) Items (Cont.)

- Consider the item

$$E \rightarrow E + (\,|\, E) \;, +$$

- We expect a string derived from $E\ ) +$
- Our example has two productions for E

$$E \rightarrow \text{int} \quad \text{and} \quad E \rightarrow E + (E)$$

- We describe this by extending the context with two more items:

$$E \rightarrow \,|\, \text{int} \qquad\qquad ,)$$
$$E \rightarrow \,|\, E + (E) \;,)$$

The Closure Operation

- The operation of extending the context with items is called the closure operation.

```
Closure(Items) =
  repeat
    for each [X → α | Yβ, a] in Items
      for each production Y → γ
        for each b in First(βa)
          add [Y → | γ, b] to Items
  until Items is unchanged
```

Constructing the Parsing DFA (1)

$E \rightarrow E + (E) \mid \text{int}$

- Construct the start context:

  Closure($\{S \rightarrow \mid E, \$\}$)

$$
\begin{array}{l}
S \rightarrow \mid E \quad , \$ \\
E \rightarrow \mid E+(E), \$ \\
E \rightarrow \mid \text{int} \quad , \$ \\
E \rightarrow \mid E+(E), + \\
E \rightarrow \mid \text{int} \quad , +
\end{array}
$$

- We abbreviate as:

$$
\begin{array}{l}
S \rightarrow \mid E \quad , \$ \\
E \rightarrow \mid E+(E) \quad , \$/+ \\
E \rightarrow \mid \text{int} \quad , \$/+
\end{array}
$$

Constructing the Parsing DFA (2)

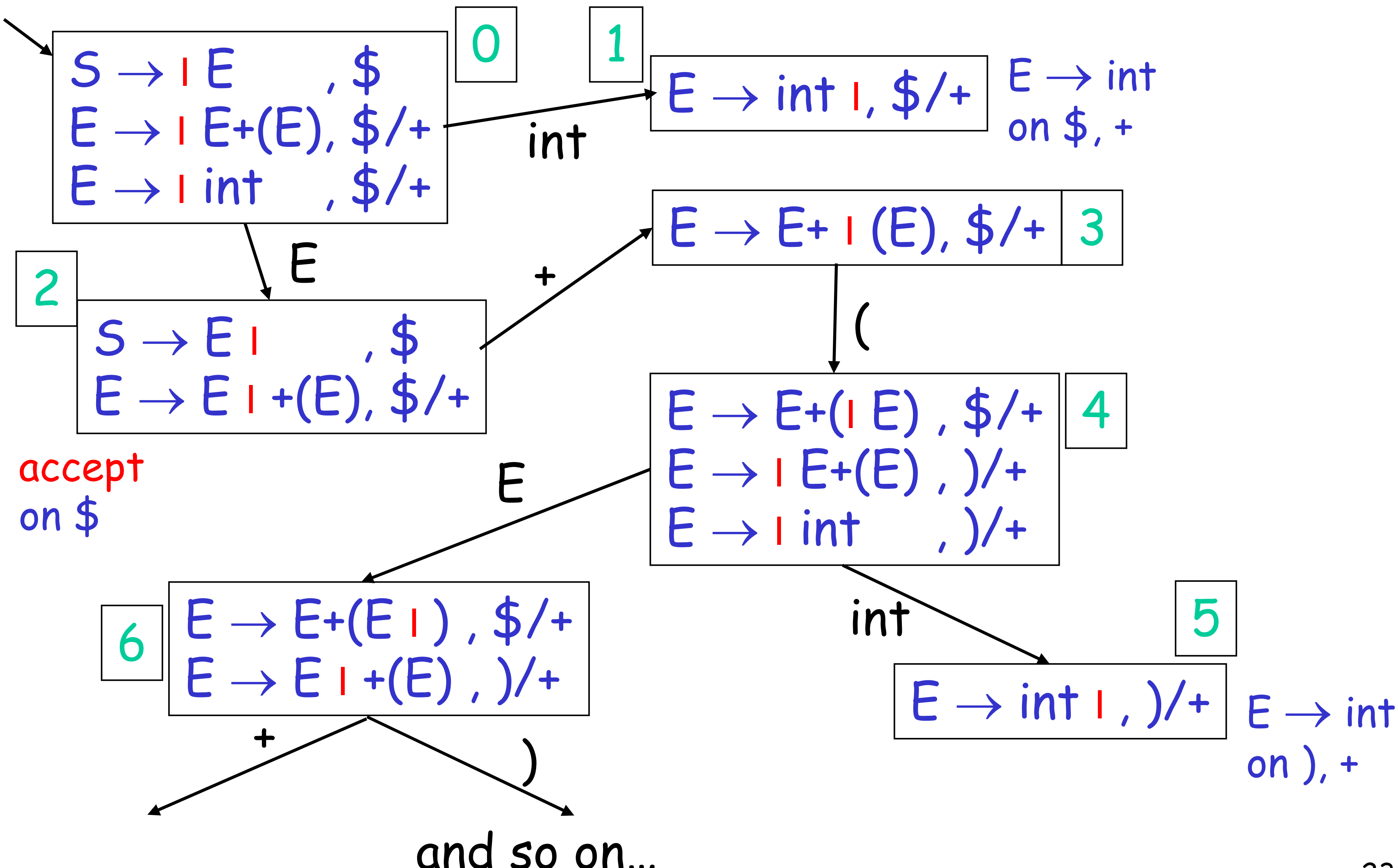
- A DFA state is a closed set of LR(1) items.
- The start state contains $[S \to \mid E \ , \$]$.
- A state that contains $[X \to \alpha \mid, b]$ is labeled with "reduce with $X \to \alpha$ on b".
- And now the transitions …

The DFA Transitions

- A state "State" that contains $[X \to \alpha \mid y\beta, b]$ has a transition labeled y to a state that contains the items "**Transition**(State, y)"
  - y can be a terminal or a non-terminal

Transition(State, y)
  Items = $\emptyset$
  for each $[X \to \alpha \mid y\beta, b]$ in State
    add $[X \to \alpha y \mid \beta, b]$ to Items
  return Closure(Items)

Constructing the Parsing DFA: Example



LR Parsing Tables: Notes

- Parsing tables (i.e., the DFA) can be constructed automatically for a CFG.

- But we still need to understand the construction to work with parser generators.
  - E.g., they report errors in terms of sets of items.

- What kind of errors can we expect?

Shift/Reduce Conflicts

- If a DFA state contains both

  $[X \rightarrow \alpha \,|\, a\beta,\ b]$ and $[Y \rightarrow \gamma \,|,\ a]$

- Then on input "a" we could either
  - Shift into state $[X \rightarrow \alpha a \,|\, \beta,\ b]$, or
  - Reduce with $Y \rightarrow \gamma$

- This is called a *shift-reduce conflict*

Shift/Reduce Conflicts

- Typically due to ambiguities in the grammar.
- Classic example: the dangling else

  $S \rightarrow$ if E then S | if E then S else S | OTHER

- We will have a DFA state containing:

  [$S \rightarrow$ if E then S $\bullet$, else]

  [$S \rightarrow$ if E then S $\bullet$ else S, x]

- If else follows then we can shift or reduce.
- Default (yacc, ML-yacc, bison, etc.) is to shift.
  - Default behavior is as needed in this case.

More Shift/Reduce Conflicts

- Consider the ambiguous grammar:

$$E \rightarrow E + E \mid E * E \mid \text{int}$$

- We will have the states containing:

$$[E \rightarrow E * \,|\, E,\ +] \qquad\qquad [E \rightarrow E * E\,|,\ +]$$

$$[E \rightarrow \,|\, E + E,\ +] \quad \Rightarrow^{E} \quad [E \rightarrow E \,|\, + E,\ +]$$

$$\ldots \qquad\qquad\qquad\qquad \ldots$$

- Again we have a shift/reduce on input +
  - We need to reduce (* binds more tightly than +)
  - Recall solution: declare the precedence of * and +

More Shift/Reduce Conflicts

- In yacc declare precedence and associativity:

```
%left +
%left *
```

- Precedence of a rule = that of its last terminal.

  See yacc manual for ways to override this default.

- Resolve shift/reduce conflict with a <u>shift</u> if:
  - no precedence declared for either rule or terminal;
  - input terminal has higher precedence than the rule;
  - the precedences are the same and right associative.

Using Precedence to Solve S/R Conflicts

- Back to our example:

$[E \to E * \mid E,\ +]$ $\qquad$ $[E \to E * E \mid,\ +]$

$[E \to \mid E + E,\ +] \Rightarrow^{E}$ $[E \to E \mid + E,\ +]$

… $\qquad$ …

- We will choose reduce because precedence of rule $E \to E * E$ is higher than of terminal $+$.

Using Precedence to Solve S/R Conflicts

- Same grammar as before:

$$E \rightarrow E + E \mid E * E \mid \text{int}$$

- We will also have the states:

$$[E \rightarrow E + \mathbf{I}\, E,\ \ +] \qquad\qquad [E \rightarrow E + E\, \mathbf{I},\ \ +]$$

$$[E \rightarrow \mathbf{I}\, E + E,\ \ +] \quad \Rightarrow^{E} \quad [E \rightarrow E\, \mathbf{I} + E,\ \ +]$$

$$\ldots \qquad\qquad\qquad\qquad \ldots$$

- Now we also have a shift/reduce on input +
  - We will choose reduce because $E \rightarrow E + E$ and + have the same precedence and + is left-associative.

Using Precedence to Solve S/R Conflicts

- Back to our dangling else example:

  [S → if E then S •, else]

  [S → if E then S • else S, x]

- Can eliminate conflict by declaring else having higher precedence than then.
- But this starts to look like "hacking the tables".
- Best to avoid overuse of precedence declarations or we will end with unexpected parse trees.

Precedence Declarations Revisited

The term "precedence declaration" is misleading!

These declarations do not define precedence; instead, they define conflict resolutions.

- I.e., they instruct shift-reduce parsers to resolve conflicts in certain ways.
- These two are not quite the same!

Reduce/Reduce Conflicts

- If a DFA state contains both

  $$[X \rightarrow \alpha\,\cdot, a] \text{ and } [Y \rightarrow \beta\,\cdot, a]$$

  - Then on input "a" we do not know which production to reduce.

- This is called a *reduce/reduce conflict*

Reduce/Reduce Conflicts

- Usually due to gross ambiguity in the grammar.
- Ex. A grammar for a sequence of identifiers:

$$S \rightarrow \varepsilon \mid id \mid id\ S$$

- There are two parse trees for the string id

$$S \rightarrow id$$

$$S \rightarrow id\ S \rightarrow id$$

- How does this confuse the parser?

More on Reduce/Reduce Conflicts

- Consider the states

$$
\begin{array}{lll}
[S' \rightarrow \bullet\, S, \quad \$] & & [S \rightarrow id\, \bullet, \quad \$] \\
[S \rightarrow \bullet, \quad \$] & & [S \rightarrow id\, \bullet\, S, \quad \$] \\
[S \rightarrow \bullet\, id, \quad \$] & \Rightarrow^{id} & [S \rightarrow \bullet, \quad \$] \\
[S \rightarrow \bullet\, id\, S, \quad \$] & & [S \rightarrow \bullet\, id, \quad \$] \\
 & & [S \rightarrow \bullet\, id\, S, \quad \$]
\end{array}
$$

- Reduce/reduce conflict on input $\$$

$$S' \rightarrow S \rightarrow id$$

$$S' \rightarrow S \rightarrow id\ S \rightarrow id$$

- Better to rewrite the grammar as: $S \rightarrow \varepsilon \mid id\ S$

Using Parser Generators

- Parser generators automatically construct the parsing DFA given a CFG.
  - Use precedence declarations and default conventions to resolve conflicts.
  - The parser algorithm is the same for all grammars (and is provided as a library function).
- But most parser generators do not construct the DFA as described before.
  - Because the LR(1) parsing DFA has 1000s of states even for a simple language.

LR(1) Parsing Tables are Big

- But many states are similar, e.g.

1			5	
E → int •, $/+	E → int on $, +	and	E → int •,)/+	E → int on), +

- Idea: merge the DFA states whose items differ only in the lookahead tokens
  - We say that such states have the same core

- We obtain

1'	
E → int •, $/+/)	E → int on $, +,)

The Core of a Set of LR Items

Definition: The core of a set of LR items is the set of first components
- Without the lookahead terminals

- Example: the core of

$$\{[X \to \alpha \,\mathrm{I}\, \beta,\ b],\ [Y \to \gamma \,\mathrm{I}\, \delta,\ d]\}$$

is

$$\{X \to \alpha \,\mathrm{I}\, \beta,\ Y \to \gamma \,\mathrm{I}\, \delta\}$$

LALR States

- Consider for example the LR(1) states

$$\{[X \to \alpha \,\mathsf{I}, a], [Y \to \beta \,\mathsf{I}, c]\}$$

$$\{[X \to \alpha \,\mathsf{I}, b], [Y \to \beta \,\mathsf{I}, d]\}$$

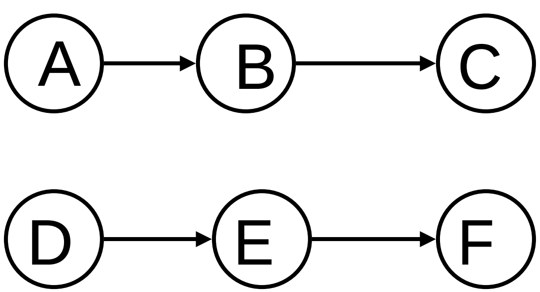
- They have the same core and can be merged
- The merged state contains:

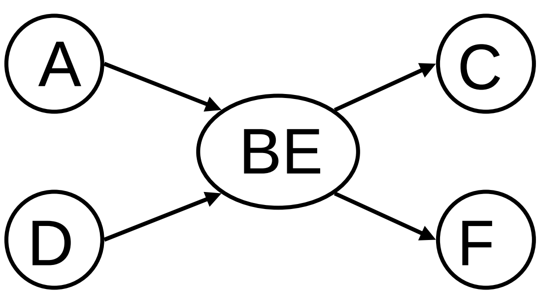
$$\{[X \to \alpha \,\mathsf{I}, a/b], [Y \to \beta \,\mathsf{I}, c/d]\}$$

- These are called LALR(1) states
  - Stands for LookAhead LR
  - Typically 10 times fewer LALR(1) states than LR(1)

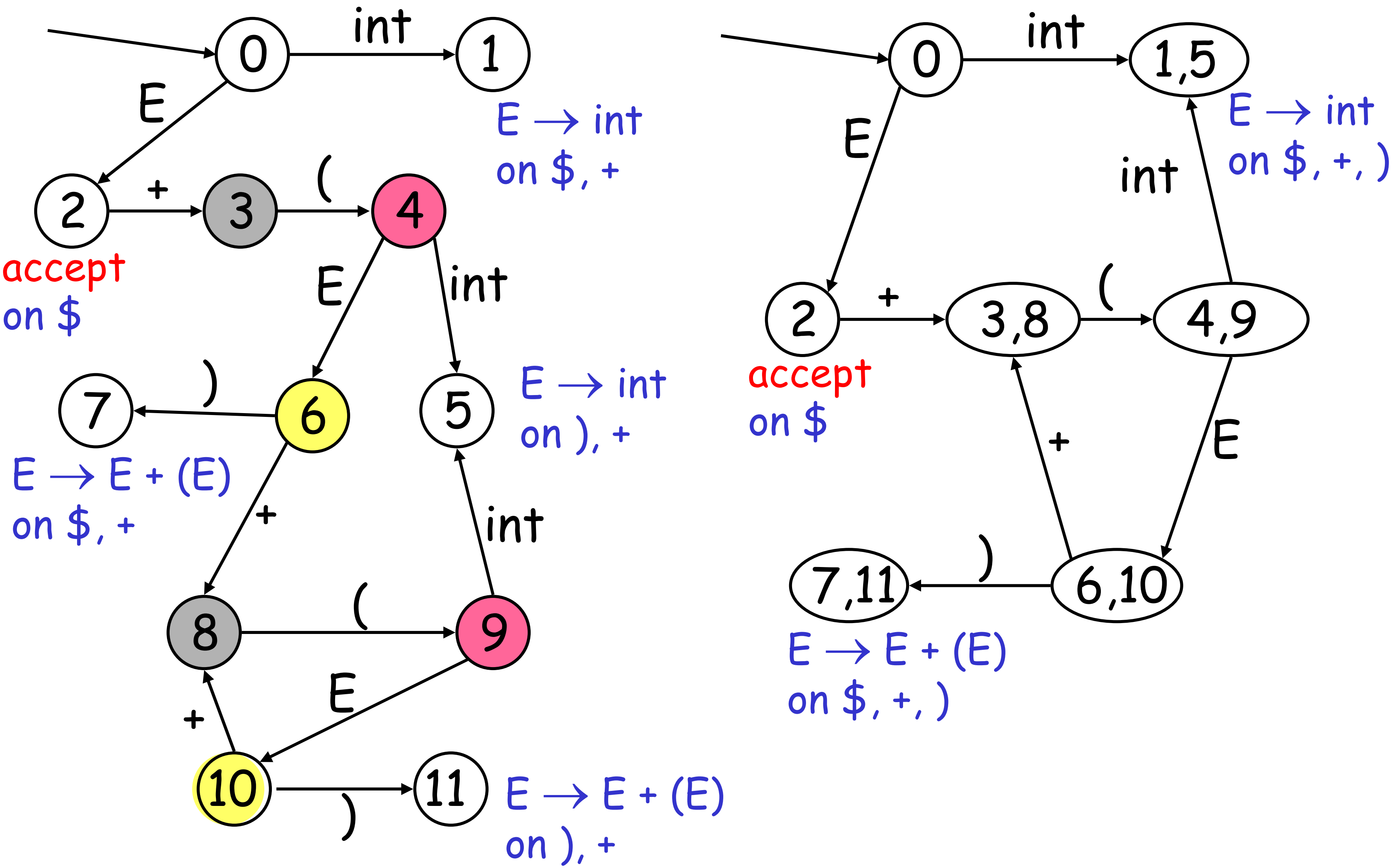
A LALR(1) DFA

- Repeat until all states have distinct core
  - Choose two distinct states with same core
  - Merge the states by creating a new one with the union of all the items
  - Point edges from predecessors to new state
  - New state points to all the previous successors





Conversion LR(1) to LALR(1): Example.



The LALR Parser Can Have Conflicts

- Consider for example the LR(1) states:

$$\{[X \to \alpha\,|, a], [Y \to \beta\,|, b]\}$$

$$\{[X \to \alpha\,|, b], [Y \to \beta\,|, a]\}$$

- And the merged LALR(1) state:

$$\{[X \to \alpha\,|, a/b], [Y \to \beta\,|, a/b]\}$$

- Has a <u>new</u> reduce/reduce conflict!

- In practice, such cases are rare.

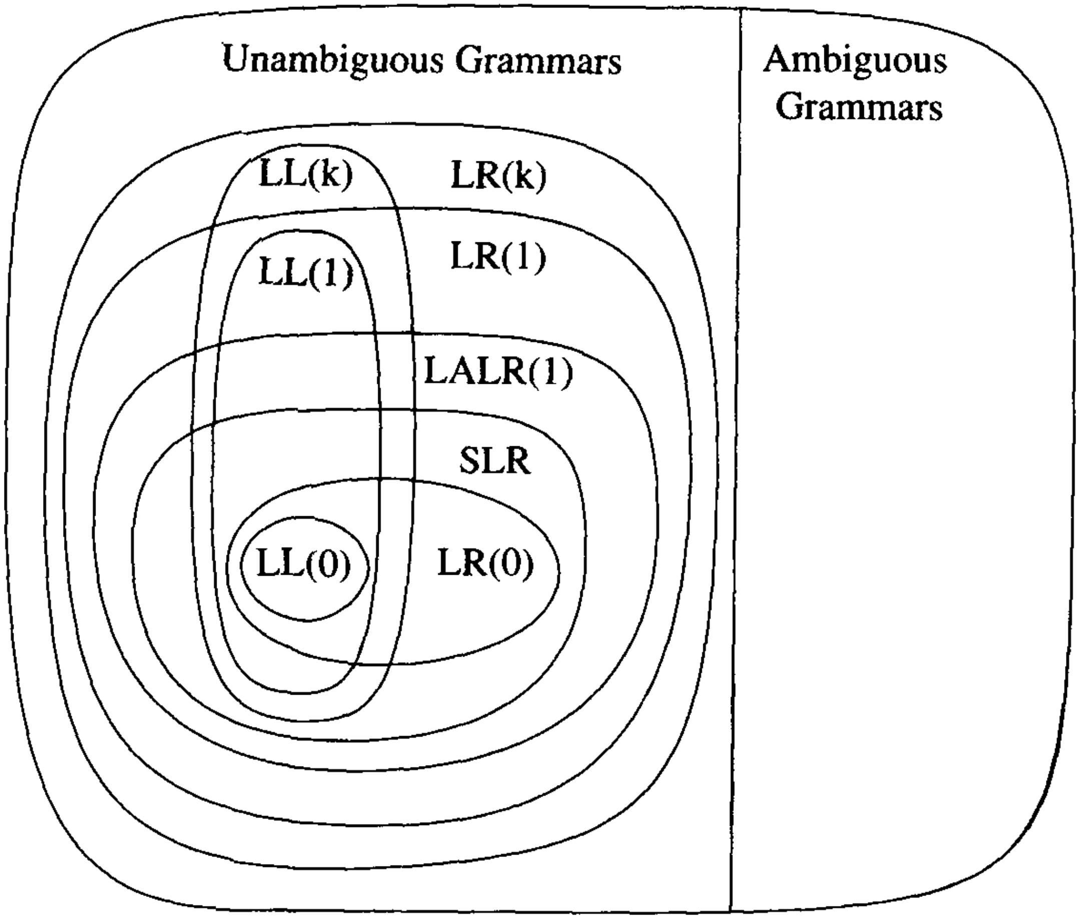
LALR vs. LR Parsing: Things to keep in mind

- LALR languages are not natural.
  - They are an "efficiency hack" on LR languages.

- Any reasonable programming language has a LALR(1) grammar.

- LALR(1) parsing has become a standard for programming languages and parser generators.

A Hierarchy of Grammar Classes



From Andrew Appel, "Modern Compiler Implementation in ML"