

LR Parsing

LALR Parser Generators

Outline

- Review of bottom-up parsing
- Computing the parsing DFA
- Using parser generators

Bottom-up Parsing (Review)

- A bottom-up parser rewrites the input string to the start symbol
- The state of the parser is described as
$$\alpha \mid \gamma$$
 - α is a stack of terminals and non-terminals
 - γ is the string of terminals not yet examined
- Initially: $\mid x_1 x_2 \dots x_n$

The Shift and Reduce Actions (Review)

Recall the CFG: $E \rightarrow E + (E) \mid \text{int}$

A bottom-up parser uses two kinds of actions:

- Shift pushes a terminal from input on the stack

$$E + (\color{red}{|} \text{int}) \Rightarrow E + (\text{int} \color{red}{|})$$

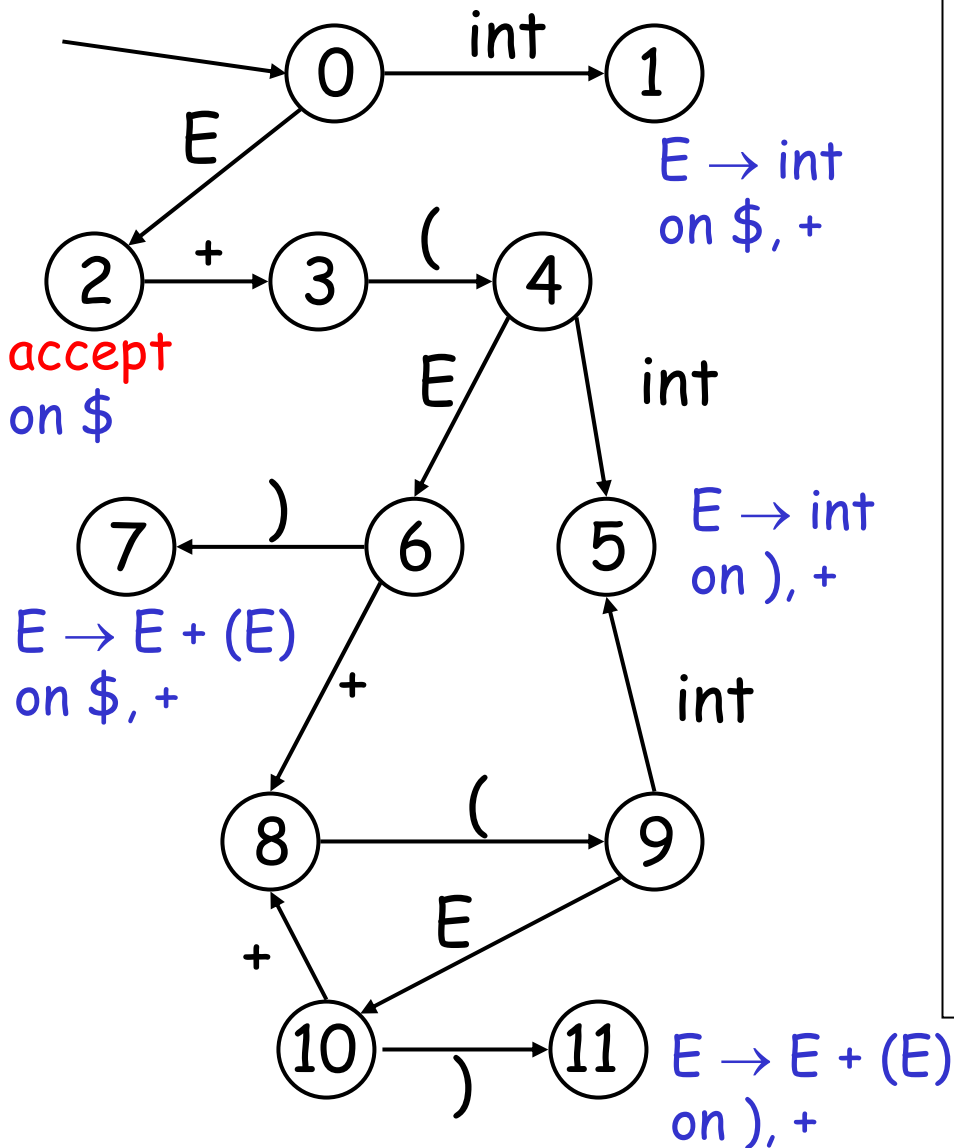
- Reduce pops 0 or more symbols off of the stack (production RHS) and pushes a non-terminal on the stack (production LHS)

$$E + (\underline{E + (E)} \color{red}{|}) \Rightarrow E + (\underline{E} \color{red}{|})$$

Key Issue: When to Shift or Reduce?

- Idea: use a deterministic finite automaton (DFA) to decide when to shift or reduce
 - The input is the stack
 - The language consists of terminals and non-terminals
- We run the DFA on the stack and we examine the resulting state X and the token tok after $|$
 - If X has a transition labeled tok then shift
 - If X is labeled with " $A \rightarrow \beta$ on tok " then reduce

LR(1) Parsing: An Example



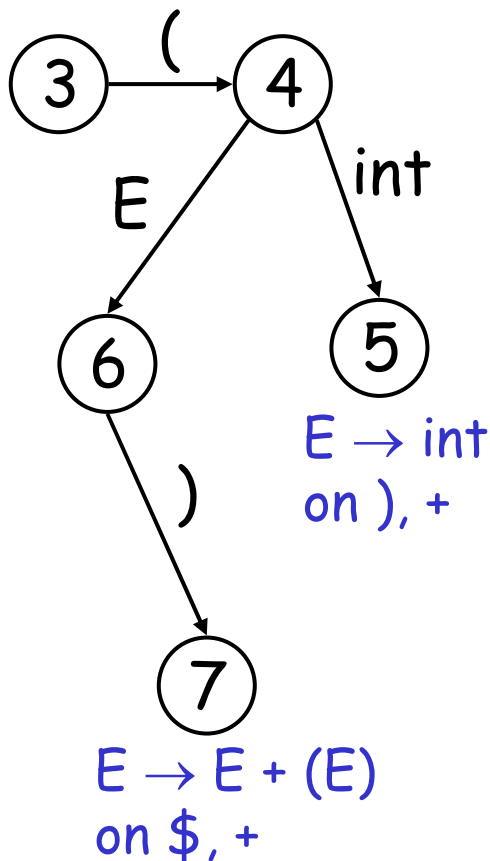
int + (int) + (int)\$	shift
int + (int) + (int)\$	$E \rightarrow \text{int}$
E + (int) + (int)\$	shift (x3)
E + (int) + (int)\$	$E \rightarrow \text{int}$
E + (E) + (int)\$	shift
E + (E) + (int)\$	$E \rightarrow E+(E)$
E + (int)\$	shift (x3)
E + (int)\$	$E \rightarrow \text{int}$
E + (E)\$	shift
E + (E) \$	$E \rightarrow E+(E)$
E \$	accept

Representing the DFA

- Parsers represent the DFA as a 2D table
(Recall table-driven lexical analysis)
- Lines correspond to DFA states
- Columns correspond to terminals and non-terminals
- Typically columns are split into:
 - Those for terminals: the **action** table
 - Those for non-terminals: the **goto** table

Representing the DFA: Example

The table for a fragment of our DFA:



	int	+	()	\$	E
...						
3				s4		
4	s5					g6
5		$r_{E \rightarrow \text{int}}$		$r_{E \rightarrow \text{int}}$		
6	s8			s7		
7		$r_{E \rightarrow E+(E)}$			$r_{E \rightarrow E+(E)}$	
...						

sk is shift and goto state k
 $r_{x \rightarrow \alpha}$ is reduce
 gk is goto state k

The LR Parsing Algorithm

- After a shift or reduce action we rerun the DFA on the entire stack
 - This is wasteful, since most of the work is repeated
- To avoid this, we remember for each stack element on which state it brings the DFA
- LR parser maintains a stack
$$\langle \text{sym}_1, \text{state}_1 \rangle \dots \langle \text{sym}_n, \text{state}_n \rangle$$
$$\text{state}_k \text{ is the final state of the DFA on } \text{sym}_1 \dots \text{sym}_k$$

The LR Parsing Algorithm

```
let I = w$ be initial input
let j = 0
let DFA state 0 be the start state
let stack = ⟨ dummy, 0 ⟩
repeat
  case action[top_state(stack), I[j]] of
    shift k: push ⟨ I[j++], k ⟩
    reduce X → A:
      pop |A| pairs,
      push ⟨ X, goto[top_state(stack), X] ⟩
    accept: halt normally
    error: halt and report error
```

Key Issue: How is the DFA Constructed?

- The stack describes the context of the parse
 - What non-terminal we are looking for
 - What production RHS we are looking for
 - What we have seen so far from the RHS
- Each DFA state describes several such contexts
 - E.g., when we are looking for non-terminal E , we might be looking either for an int or an $E + (E)$ RHS

LR(0) Items

- An LR(0) item is a production with a "**|**" somewhere on the RHS
- The LR(0) items for $T \rightarrow (E)$ are
 - $T \rightarrow | (E)$
 - $T \rightarrow (| E)$
 - $T \rightarrow (E |)$
 - $T \rightarrow (E) |$
- The only LR(0) item for $X \rightarrow \varepsilon$ is $X \rightarrow |$

LR(0) Items: Intuition

- An item $[X \rightarrow \alpha \mid \beta]$ says that the parser
 - is looking for an X
 - has an α on top of the stack
 - expects to find a string derived from β next in the input
- Notes:
 - $[X \rightarrow \alpha \mid a\beta]$ means that a should follow
 - Then we can shift it and still have a viable prefix
 - $[X \rightarrow \alpha \mid]$ means that we could reduce X
 - But this is not always a good idea !

LR(1) Items

- An LR(1) item is a pair:
 - $X \rightarrow \alpha \mid \beta, a$
 - $X \rightarrow \alpha\beta$ is a production
 - a is a terminal (the lookahead terminal)
 - LR(**1**) means **1** lookahead terminal
- $[X \rightarrow \alpha \mid \beta, a]$ describes a context of the parser
 - We are trying to find an X followed by an a , and
 - We have (at least) α already on top of the stack
 - Thus we need to see next a prefix derived from βa

Note

- The symbol $|$ was used before to separate the stack from the rest of input
 - $\alpha | \gamma$, where α is the stack and γ is the remaining string of terminals
- In items, $|$ is used to mark a prefix of a production RHS:
$$X \rightarrow \alpha | \beta, a$$
 - Here β might contain non-terminals as well
- In either case the stack is on the left of $|$

Convention

- We add to our grammar a fresh new start symbol S and a production $S \rightarrow E$
 - Where E is the old start symbol
- The initial parsing context contains:
$$S \rightarrow | E , \$$$
 - Trying to find an S as a string derived from $E\$$
 - The stack is empty

LR(1) Items (Cont.)

- In context containing

$$E \rightarrow E + | (E) , +$$

- If (follows then we can perform a shift to context containing

$$E \rightarrow E + (| E) , +$$

- In context containing

$$E \rightarrow E + (E) | , +$$

- We can perform a reduction with $E \rightarrow E + (E)$
- But only if a + follows

LR(1) Items (Cont.)

- Consider the item

$$E \rightarrow E + (\mid E) , +$$

- We expect a string derived from $E) +$
- Our example has two productions for E

$$E \rightarrow \text{int} \quad \text{and} \quad E \rightarrow E + (E)$$

- We describe this by extending the context with two more items:

$$E \rightarrow \mid \text{int} \quad ,)$$

$$E \rightarrow \mid E + (E) ,)$$

The Closure Operation

- The operation of extending the context with items is called the closure operation

```
Closure(Items) =  
  repeat  
    for each  $[X \rightarrow \alpha \mid Y\beta, a]$  in Items  
      for each production  $Y \rightarrow \gamma$   
        for each  $b$  in  $\text{First}(\beta a)$   
          add  $[Y \rightarrow \mid \gamma, b]$  to Items  
  until Items is unchanged
```

Constructing the Parsing DFA (1)

- Construct the start context:

$E \rightarrow E + (E) \mid \text{int}$

Closure($\{S \rightarrow \mid E, \$\}$)

$S \rightarrow \mid E, \$$

$E \rightarrow \mid E+(E), \$$

$E \rightarrow \mid \text{int}, \$$

$E \rightarrow \mid E+(E), +$

$E \rightarrow \mid \text{int}, +$

- We abbreviate as:

$S \rightarrow \mid E, \$$

$E \rightarrow \mid E+(E), \$/+$

$E \rightarrow \mid \text{int}, \$/+$

Constructing the Parsing DFA (2)

- A DFA state is a closed set of LR(1) items
- The start state contains $[S \rightarrow | E, \$]$
- A state that contains $[X \rightarrow \alpha |, b]$ is labeled with "reduce with $X \rightarrow \alpha$ on b "
- And now the transitions ...

The DFA Transitions

- A state "State" that contains $[X \rightarrow \alpha \mid y\beta, b]$ has a transition labeled y to a state that contains the items "Transition(State, y)"
 - y can be a terminal or a non-terminal

Transition(State, y)

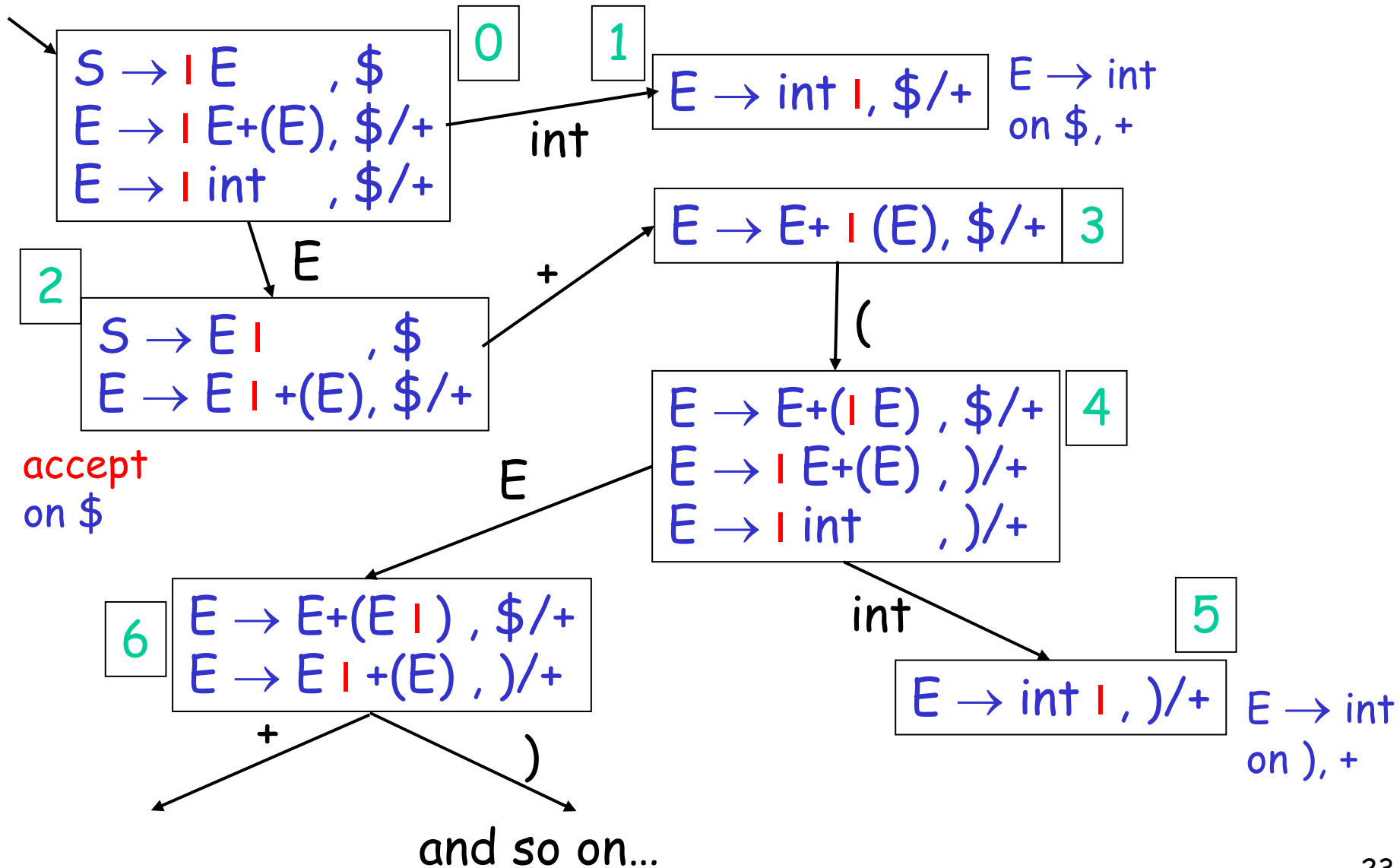
Items = \emptyset

for each $[X \rightarrow \alpha \mid y\beta, b]$ in State

add $[X \rightarrow \alpha y \mid \beta, b]$ to Items

return Closure(Items)

Constructing the Parsing DFA: Example



LR Parsing Tables: Notes

- Parsing tables (i.e., the DFA) can be constructed automatically for a CFG
- But we still need to understand the construction to work with parser generators
 - E.g., they report errors in terms of sets of items
- What kind of errors can we expect?

Shift/Reduce Conflicts

- If a DFA state contains both
 $[X \rightarrow \alpha \mid a\beta, b]$ and $[Y \rightarrow \gamma \mid, a]$
- Then on input "a" we could either
 - Shift into state $[X \rightarrow \alpha a \mid \beta, b]$, or
 - Reduce with $Y \rightarrow \gamma$
- This is called a *shift-reduce conflict*

Shift/Reduce Conflicts

- Typically due to ambiguities in the grammar
- Classic example: the **dangling else**
 $S \rightarrow \text{if } E \text{ then } S \mid \text{if } E \text{ then } S \text{ else } S \mid \text{OTHER}$
- Will have DFA state containing
 $[S \rightarrow \text{if } E \text{ then } S \mid, \quad \text{else}]$
 $[S \rightarrow \text{if } E \text{ then } S \mid \text{else } S, \quad x]$
- If **else** follows then we can shift or reduce
- Default (**yacc**, **ML-yacc**, **bison**, etc.) is to shift
 - Default behavior is as needed in this case

More Shift/Reduce Conflicts

- Consider the ambiguous grammar

$$E \rightarrow E + E \mid E * E \mid \text{int}$$

- We will have the states containing

$$\begin{array}{cc} [E \rightarrow E * \mid E, +] & [E \rightarrow E * E \mid, +] \\ [E \rightarrow \mid E + E, +] & \Rightarrow^E [E \rightarrow E \mid + E, +] \\ \dots & \dots \end{array}$$

- Again we have a shift/reduce on input +
 - We need to reduce (* binds more tightly than +)
 - Recall solution: declare the precedence of * and +

More Shift/Reduce Conflicts

- In `yacc` declare precedence and associativity:
 `%left +`
 `%left *`
- Precedence of a rule = that of its last terminal
 See `yacc` manual for ways to override this default
- Resolve shift/reduce conflict with a shift if:
 - no precedence declared for either rule or terminal
 - input terminal has higher precedence than the rule
 - the precedences are the same and right associative

Using Precedence to Solve S/R Conflicts

- Back to our example:

$$\begin{array}{cc} [E \rightarrow E * | E, +] & [E \rightarrow E * E |, +] \\ [E \rightarrow | E + E, +] \Rightarrow^E & [E \rightarrow E | + E, +] \\ \dots & \dots \end{array}$$

- Will choose reduce because precedence of rule $E \rightarrow E * E$ is higher than of terminal $+$

Using Precedence to Solve S/R Conflicts

- Same grammar as before

$$E \rightarrow E + E \mid E * E \mid \text{int}$$

- We will also have the states

$$[E \rightarrow E + \mid E, +] \qquad [E \rightarrow E + E \mid, +]$$

$$[E \rightarrow \mid E + E, +] \Rightarrow^E [E \rightarrow E \mid + E, +]$$

...

...

- Now we also have a shift/reduce on input +
 - We choose reduce because $E \rightarrow E + E$ and $+$ have the same precedence and $+$ is left-associative

Using Precedence to Solve S/R Conflicts

- Back to our dangling else example
 - [S → if E then S |, else]
 - [S → if E then S | else S, x]
- Can eliminate conflict by declaring **else** having higher precedence than **then**
- But this starts to look like “hacking the tables”
- Best to avoid overuse of precedence declarations or we will end with unexpected parse trees

Precedence Declarations Revisited

The term “precedence declaration” is misleading!

These declarations do not define precedence:
they define conflict resolutions

I.e., they instruct shift-reduce parsers to resolve
conflicts in certain ways

These two are not quite the same!

Reduce/Reduce Conflicts

- If a DFA state contains both
 $[X \rightarrow \alpha \mid, a]$ and $[Y \rightarrow \beta \mid, a]$
 - Then on input "a" we don't know which production to reduce
- This is called a *reduce/reduce conflict*

Reduce/Reduce Conflicts

- Usually due to gross ambiguity in the grammar
- Example: a sequence of identifiers

$$S \rightarrow \varepsilon \mid id \mid id S$$

- There are two parse trees for the string `id`

$$S \rightarrow id$$

$$S \rightarrow id S \rightarrow id$$

- How does this confuse the parser?

More on Reduce/Reduce Conflicts

- Consider the states

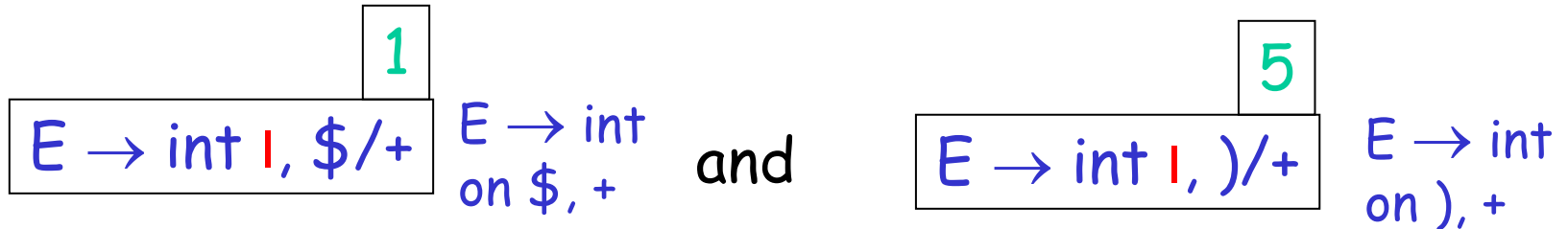
$[S' \rightarrow S, \$]$	$[S \rightarrow id , \$]$	\Rightarrow^{id}	$[S \rightarrow id S, \$]$
$[S \rightarrow , \$]$			$[S \rightarrow , \$]$
$[S \rightarrow id, \$]$			$[S \rightarrow id, \$]$
$[S \rightarrow id S, \$]$			$[S \rightarrow id S, \$]$
- Reduce/reduce conflict on input \$
 $S' \rightarrow S \rightarrow id$
 $S' \rightarrow S \rightarrow id S \rightarrow id$
- Better to rewrite the grammar as: $S \rightarrow \varepsilon \mid id S$

Using Parser Generators

- Parser generators automatically construct the parsing DFA given a CFG
 - Use precedence declarations and default conventions to resolve conflicts
 - The parser algorithm is the same for all grammars (and is provided as a library function)
- But most parser generators do not construct the DFA as described before
 - Because the LR(1) parsing DFA has 1000s of states even for a simple language

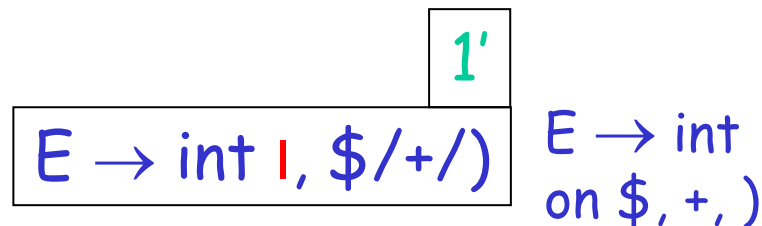
LR(1) Parsing Tables are Big

- But many states are similar, e.g.



- Idea: merge the DFA states whose items differ only in the lookahead tokens
 - We say that such states have the same core

- We obtain



The Core of a Set of LR Items

Definition: The core of a set of LR items is the set of first components

- Without the lookahead terminals

• Example: the core of

$\{[X \rightarrow \alpha \mid \beta, b], [Y \rightarrow \gamma \mid \delta, d]\}$

is

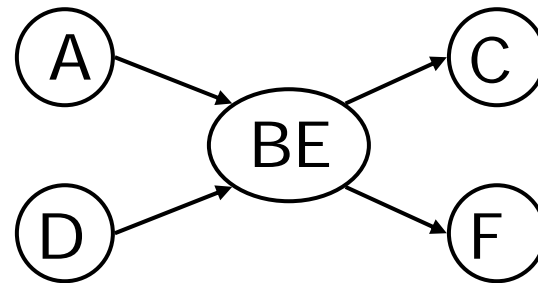
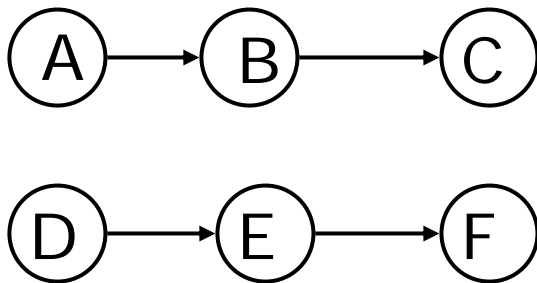
$\{X \rightarrow \alpha \mid \beta, Y \rightarrow \gamma \mid \delta\}$

LALR States

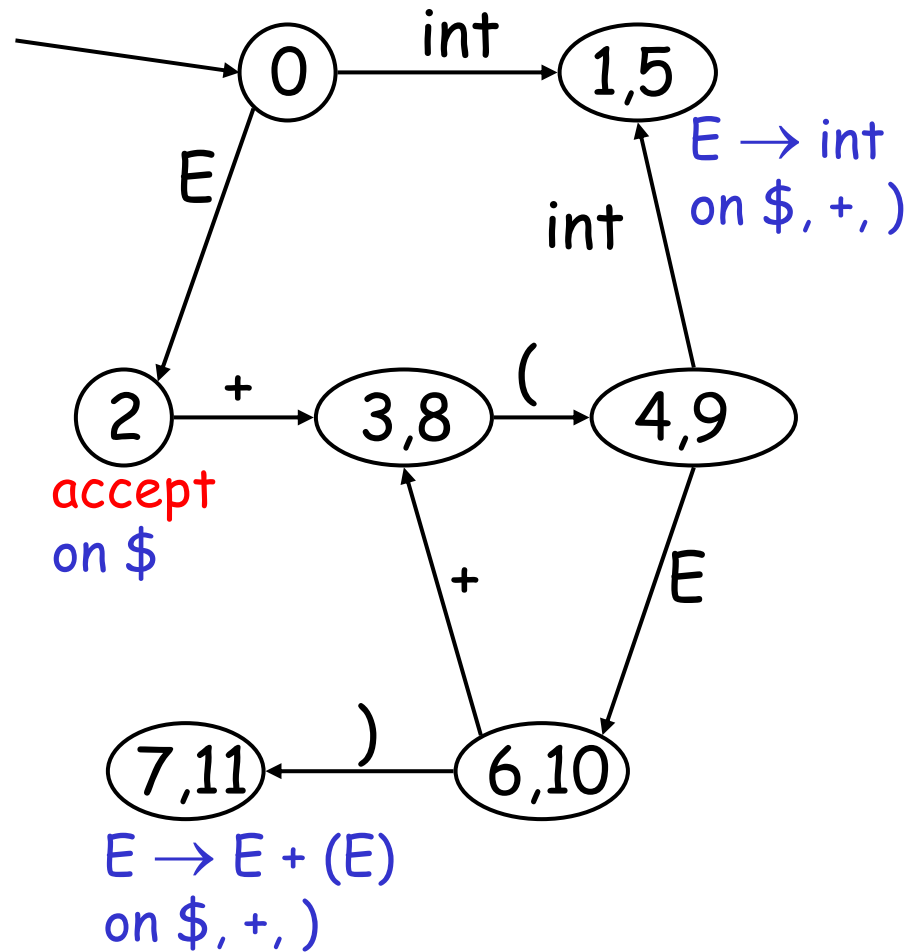
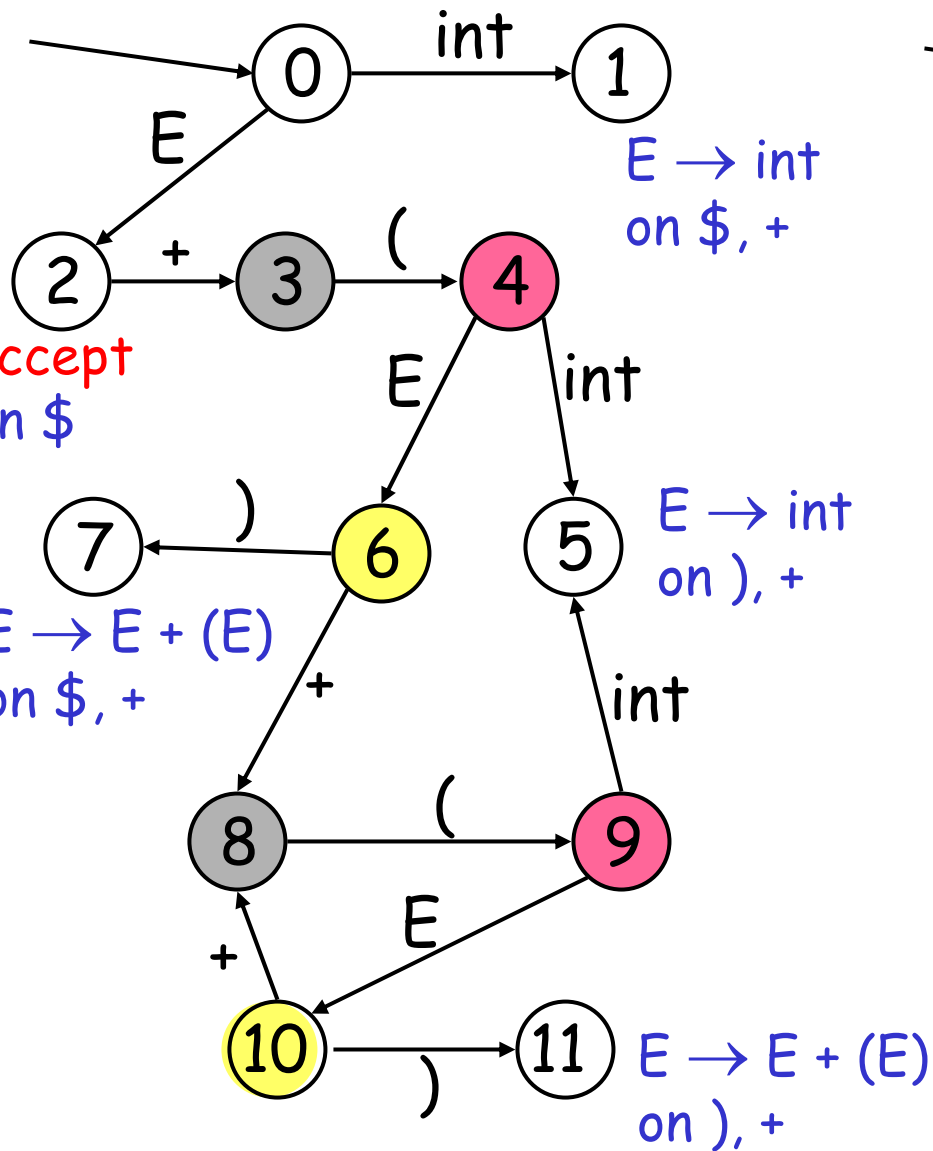
- Consider for example the LR(1) states
$$\{[X \rightarrow \alpha \mid, a], [Y \rightarrow \beta \mid, c]\}$$
$$\{[X \rightarrow \alpha \mid, b], [Y \rightarrow \beta \mid, d]\}$$
- They have the same core and can be merged
- The merged state contains:
$$\{[X \rightarrow \alpha \mid, a/b], [Y \rightarrow \beta \mid, c/d]\}$$
- These are called **LALR(1)** states
 - Stands for **L**ook**A**head **LR**
 - Typically 10 times fewer LALR(1) states than LR(1)

A LALR(1) DFA

- Repeat until all states have distinct core
 - Choose two distinct states with same core
 - Merge the states by creating a new one with the union of all the items
 - Point edges from predecessors to new state
 - New state points to all the previous successors



Conversion LR(1) to LALR(1): Example.



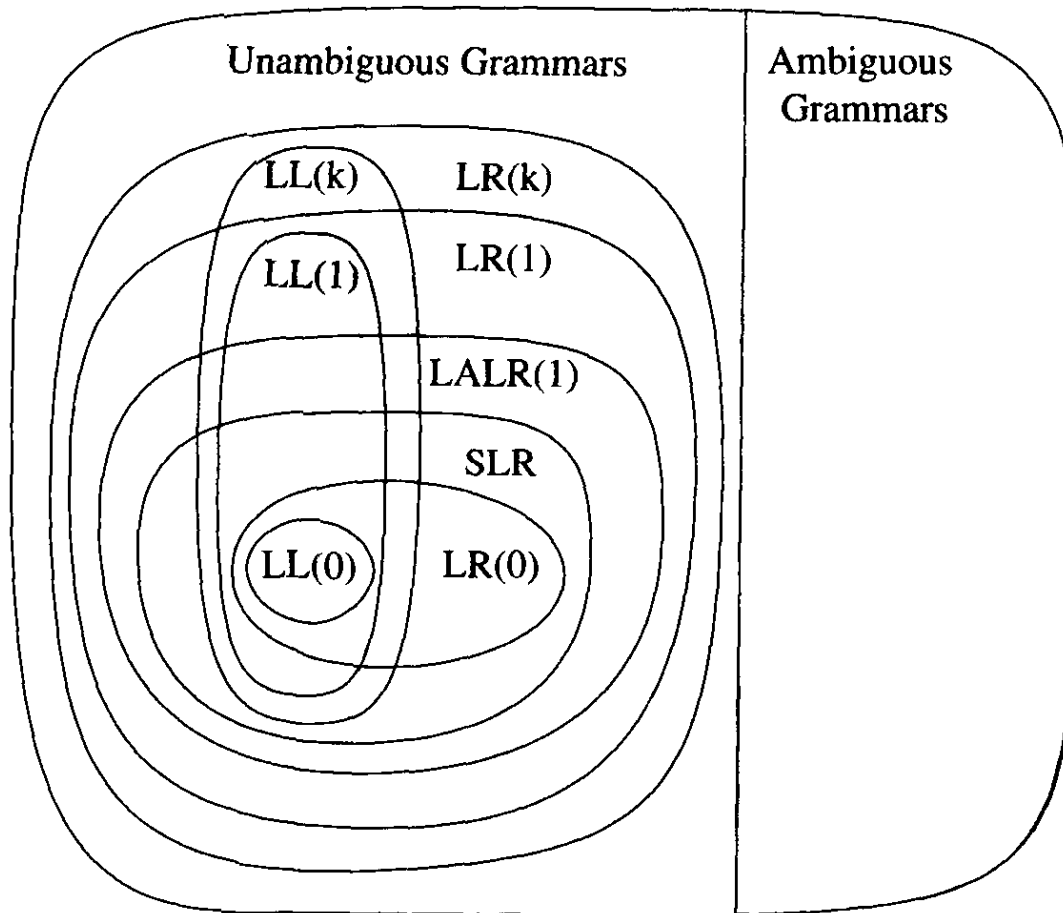
The LALR Parser Can Have Conflicts

- Consider for example the LR(1) states
$$\{[X \rightarrow \alpha \mid, a], [Y \rightarrow \beta \mid, b]\}$$
$$\{[X \rightarrow \alpha \mid, b], [Y \rightarrow \beta \mid, a]\}$$
- And the merged LALR(1) state
$$\{[X \rightarrow \alpha \mid, a/b], [Y \rightarrow \beta \mid, a/b]\}$$
- Has a new reduce/reduce conflict
- In practice such cases are rare

LALR vs. LR Parsing: Things to keep in mind

- LALR languages are not natural
 - They are an efficiency hack on LR languages
- Any reasonable programming language has a LALR(1) grammar
- LALR(1) parsing has become a standard for programming languages and parser generators

A Hierarchy of Grammar Classes



From Andrew Appel,
"Modern Compiler
Implementation in ML"